SENSITIVITY STUDIES IN PROFITABILITY ANALYSIS

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by

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DEDICATION

This thesis is dedicated to the author's wife, Linda. Without her untiring help, dedication and understanding, this work would not have been possible.

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ABSTRACT

In this thesis a profitability model is developed for an exploratory well drilling program. The model considers the number of wells drilled, the probability of success, the reserve size discovered by a successful well, the discounted profit per barrel for oil produced and the cost of an unsuccessful well or "dry hole". Each of these parameters are analyzed and a probability distribution developed for all but the dry hole costs. With this model and the respective probability distributions, a FORTRAN IV computer program is developed using the Monte Carlo technique of stochastic simulation. The program provides the best estimate of discounted profit for the venture, the risk of the venture as measured by the standard deviation (and the coefficient of variation) and the probability distribution of the discounted profit - all after any number of prespecified model executions.

After developing a hypothetical set of base parameter data and executing the program, a reference set of values for the profitability and risk are obtained. Each parameter set is then varied holding the remaining parameters at their base values. Using this technique the effects of parameter changes on observed model results are assessed.

Several different sensitivity studies are performed in this manner using the data thus developed. These studies include the effects of parameter changes on the expected profit and risk of the venture using a Significance Ratio technique

developed in the thesis. In addition, the convergence of the best estimate of discounted profit with the number of model executions, as well as the relationship between the number of wells drilled and the venture risk are analyzed.

Also studied is the possible effects of periodicities in the combined distribution data with an increased number of model executions and the effects of changing a parameter probability distribution on the best estimate of discounted profit for the venture.

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CHAPTER I

INTRODUCTION

1.1 General

A primary consideration of management in making investment decisions is the eventual profitability of the proposed venture. There are several ways of measuring profitability such as Rate of Return, and Present Worth or Discounted Expected Profit. The main purpose in calculating this profitability criterion is to summarize into a single measure the quantifiable factors affecting the economic desirability of the project under question [25].

Profitability calculations have generally been based on the best estimates of the individual variables involved. However, these best estimates may or may not occur. The resultant single value of profitability based on these best estimates gives no information on the chances of doing much better or much worse then the expected value.

In his article, Risk Analysis in Capital Investment [23], David B. Hertz demonstrates that in certain cases the conclusions reached on investment decisions may have been much different if the analysis provided not only a single profitability estimate but also the chances of that estimate being exceeded or, on the other hand, not being met. Hertz concludes that the application of probabilities will often yield entirely different and better decisions. Thus, by developing the probability distribution of the profitability, management can

make quantitative assessments of the risks involved in approving a particular investment. The chances of achieving a minimum desired return or of losing money can be determined so that management knows the size of the risk it is undertaking.

In order to determine the profitability of the venture and the probability distribution of the profitability, a model must be established containing all the factors which bear on the problem. In some cases the model is of such a nature that it can be solved analytically. That is, the model consists of known mathematical procedures or algorithms, which when manipulated, either directly or iteratively, provide a solution. Many types of managerial problems, however, are so complex that neither direct nor iterative analytical procedures exist for their solution, or, if such methods do exist, they represent such great difficulties in application that their utilization is impractical. In such cases we turn to the use of a technique called simulation. Richards and Greenlaw [42] discuss simulation as follows:

> Simulation involves designing and utilizing a model which replicates some aspect of the firm's operations... In general, simulation models may be distinguished from other operations research models in that they are non-analytical in nature, calling for an experimental approach, in which workable, satisficing solutions rather than optimal ones are usually provided.

In this thesis an oil exploration investment decision model will be developed incorporating several different variables, all of which bear on the profitability of an oil exploration venture.

Each variable will have a different probability distribution making the model so complex that analytical solution is not practical. We will therefore turn to simulation techniques, namely the Monte Carlo technique (described in Chapter II), to provide a measure of profitability and a probability distribution of that profitability.

Before discussing the organization of this thesis it may be well to clarify one point which seems to cause a great deal of confusion, that is, the distinction between uncertainty and risk. Contemporary literature, by and large, still maintains F.H. Knight's distinction between these two terms [29]. Risk is characterized by situation in which the outcome is not known with certainty, but where the probabilities of the outcomes are either known or can be estimated. Uncertainty, on the other hand, refers to situations for which probabilities of the outcomes cannot even be predicted in probabilistic terms.

Concerning this distinction, Alfred Rappaport [39], writes:

> While this distinction may be a useful pedagogical device, there is serious doubt about its applicability to decision makers in an organizational context. Here the distinction is blurred by the fact that decision makers generally have some feelings about the probabilities of future events. Admittedly, these feelings range from a degree of confidence bordering on certainty to ill-defined feelings approaching mysticism. The critical point remains that these feelings affect the judgments made by the decision maker.

Based on the reasons expressed by Rappaport the terms "uncertainty" and "risk" will be used interchangeably throughout this thesis.

1.2 Organization of the Thesis

This thesis treats the sensitivity studies in profitability analysis in two parts. The first part, beginning with Chapter II, describes the Monte Carlo method of stochastic simulation, a technique used throughout the thesis. In Chapter III a general profitability model for an exploratory oil drilling venture is developed. Chapter IV discribes some general statistical "yardsticks" used to measure central tendency and variability of probability distributions. Beginning with Chapter V and continuing through Chapter VIII each variable, or parameter, in the profitability model is analyzed and a probability distribution best describing the parameter is established. In addition, a hypothetical set of values for each parameter is established which will be used as a Base for the subsequent sensitivity studies. With the profitability model and the respective probability distributions established, a FORTRAN IV computer program is developed in Chapter IX using the Monte Carlo method to provide the best estimate of discounted profit for the venture, the probability distribution of the profit and the risk of the venture. Finally, in Chapter X, using the hypothetical set of values for each parameter previously developed, the model is executed and a reference, or Base, set of values are obtained for the venture profitability and risk.

The second part of the thesis deals with the actual sensitivity studies using the model, parameter probability

distributions and Base values developed in previous chapters.

Chapter XI describes the need for sensitivity analysis in general terms and sets the framework for the studies which follow. In Chapter XII the results of the sensitivity studies are reported and analyzed. Conclusions and recommendations resulting from these studies and analyses are contained in Chapters XIII and XIV, respectively.

CHAPTER II

STOCHASTIC SIMULATION - THE MONTE CARLO TECHNIQUE

2.1 History

Through the ages man has used chance processes. Theories of chance processes were known in ancient times. The evidence is in the casting of the astragal bones from the ankles of sheep and goats in the manner of dice to subdivide land holdings. This was called casting lots. However, it was not until the 17th century that probability theory became formalized. Interestingly, this mathematical development arose out of inquiries into games of chance.

Another important development occurred in the 1940's during war work on the atomic bomb. Scientists were frustrated by being unable to solve some complex nonprobabilistic mathematical problems directly by analysis. By combining chance processes and probability theory, they solved the problems indirectly. To do this, they created a new problem stated in terms of stochastic models in such a way that the new problem was equivalent to the complex nonprobabilistic problem. But the stochastic version represented chance processes that could actually be carried out. They actually performed the chance processes represented by the probabilistic, or stochastic, formulas in the new version of their problem. Their methods were equivalent to flipping coins or throwing dice, but much more efficient. The results of these experiments were then substituted for the stochastic expressions of the new version of the problem to give

answers to the original nonprobabilistic problem. These scientists gave the name Monte Carlo to their new technique [5][22].

2.2 The Monte Carlo Technique

Monte Carlo techniques were born when probability theory and actual chance processes were used to solve problems that had no stochastic aspects at all. Since that time Monte Carlo techniques have been used to solve a multitude of nonstochastic (or deterministic) type problems [33].

When dealing with stochastic (or probabilistic) simulation it is necessary to have a means for dealing with those probabilistic variables, the values which assume a frequency distribution. Richard F. Barton [4] defines stochastic simulation as:

> ... one in which differing outputs trial to trial can be obtained without changing the inputs (ignoring random numbers as inputs). Specifically, this means that identical parameters, starting conditions, and input time path values produce varying outputs trial to trial and run to run.

One commonly used method for working with stochastic variables in simulation models is again the Monte Carlo technique. The technique is one which both has application to an almost endless variety of stochastic simulation problems and at the same time, is relatively simple to comprehend and easy to utilize. S.W. Hess and H.A. Quigley [24] define the Monte Carlo technique as:

> ... a sampling procedure whereby complicated expressions involving one or more probability distributions may be evaluated. In essence, it consists of simulation an experiment to determine some probabilistic property of a population of objects or events by the use of random sampling applied to the components of the objects or events.

> A parameter value is obtained by drawing randomly from the parameter's probability distribution. In a similar manner, selections are made for each parameter value from its respective distribution. This set of parameter values is then substituted into the model and the first sample value of the independent variable is computed. Subsequent values of the independent variable are obtained by drawing additional sets of parameter values.
>
> The individual values of the independent variable so obtained are in approximation to its true probability distribution. The approximate distribution approaches the true distribution as the sample size increases.

The Monte Carlo technique actually involves a two stage process. In the first stage, stochastic relationships between the variables are determined in order to construct the simulation model. These relationships may be determined from surveys, company data, industry data, executive experience and the like. At any rate, once the relationships are determined, probabilities of occurrence of frequency distributions are assigned for each variable. The frequency distributions are then represented by associating groups of numbers to each value of the distribution, the size of the group being governed by the relative frequency of the value. As an example, assume 60% of the items in the distribution have a value A, 30% have a value B and 10% have a value C. These three values can be represented by numbers between 00 and 99. Thus the items having a value A would be represented by the numbers from 00 through 59, those having values of B and C would be represented by numbers from 60 through 89 and 90 through 99, respectively.

Once the model is developed, frequency distributions assigned and numbers representing these frequency distributions

determined, we move to stage two which involves running the Monte Carlo process to obtain a variable value. We do this by first selecting a random number (or numbers) from tables if the simulation is to be performed manually or by building a random number generator into a computerized simulation program. (With some computer systems, such as the CDC 6600, it is possible to make use of a random number generating function incorporated in the computer systen itself.) Once the random number is selected (or generated) it is matched to the corresponding number assigned to represent each value of the frequency distribution of the variable in question. This in turn establishes the value of that variable. This process is repeated for each variable in the model, the variables combined and a single answer is obtained. Stage two is then repeated as many times as is necessary to obtain a statistically reliable sample. The various stages of the Monte Carlo process have been schematically diagrammed by M.D. Richards and P.S.Greenlaw [41] and are shown in Figure 2.1.

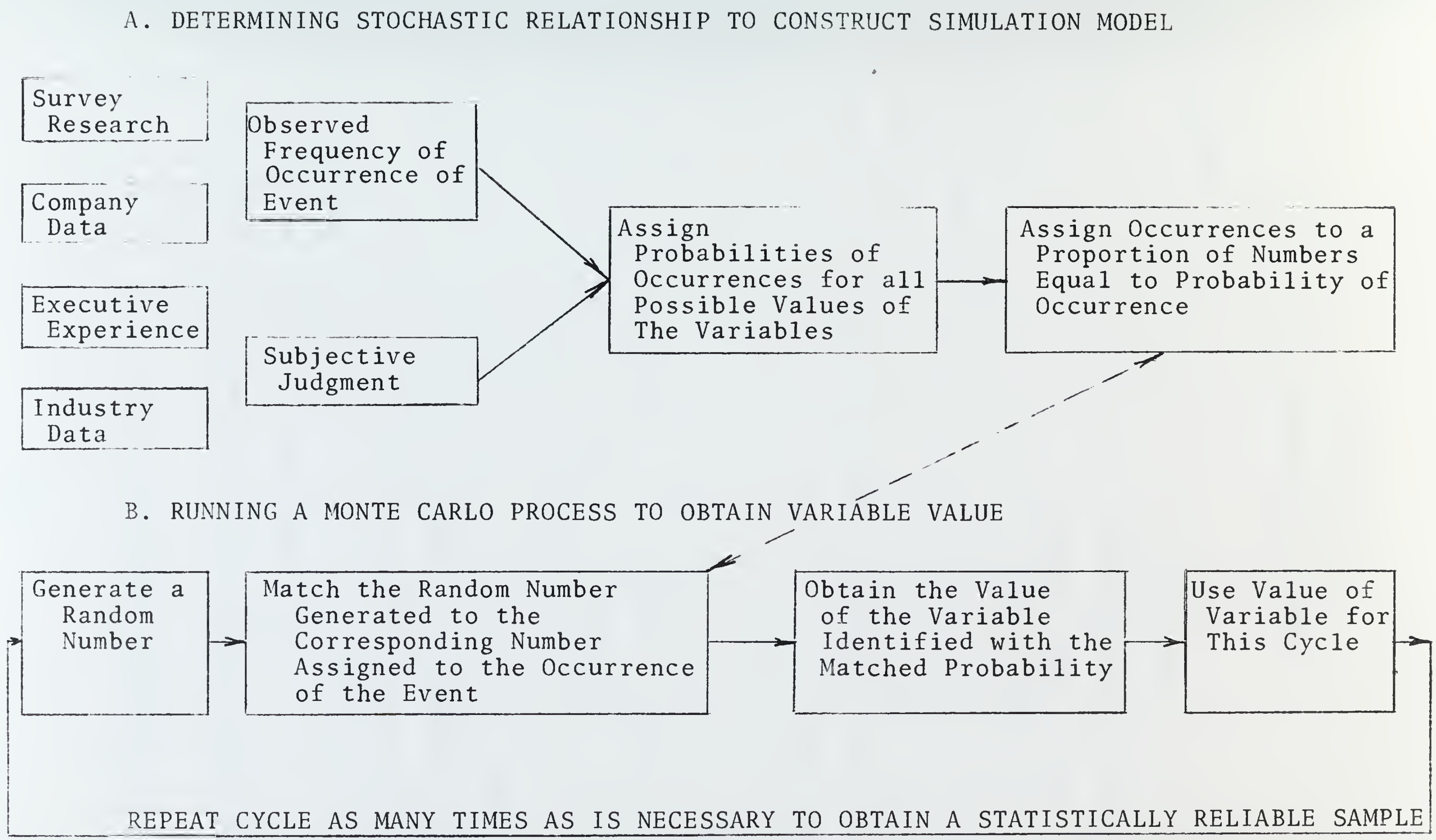


Figure 2-1

THE MONTE CARLO PROCESS

CHAPTER III

THE PROFITABILITY MODEL

3.1 General

As noted in Chapter I, the profitability of an exploratory well drilling venture and the probability distribution of that profitability will be analyzed using a model which incorporates the several different factors which bear on the problem, namely: the number of wells drilled in the program, the probability of a success, the size of field discovered by a successful well, the discounted profit per barrel of oil discovered and the cost of a dry hole, or an unsuccessful well. The model essentially measures the profitability of the venture on the basis of Net Present Worth technique whereby the expected cash flows are discounted at a suitable discount rate to reflect the time value of money. In the model, and throughout this thesis, the following notation will be utilized:

P = Total discounted profit on any one drilling endeavor.

$E(P)$ = The best estimate (mean) of discounted profit for the total exploratory well drilling venture.

n = Number of wells drilled in the drilling venture.

p = Probability of a successful well.

x = Number of successful wells, where x can assume values from o to n.

Q = Reserve size of oil field discovered by a successful well (in million bbls).

ΣQ = Reserve size of oil fields discovered in the venture (in million bbls).

e = Discounted profit per barrel for oil produced (in \$/bbl).

C_u = Cost for an unsuccessful well, or dry hole.

Using this notation, the model for a single drilling endeavor might be expressed as follows:

$$P = x\ Q\ e - (n-x)\ C_u \tag{1}$$

In an exploratory well drilling venture it must be realized that the number of successful wells is a function of both the number of wells drilled, n, and the probability of success, p. Therefore:

$$x = f(n,\ p) \tag{2}$$

Further, the size of reserve discovered in the venture, $\sum Q$, is a function of the number of successful wells.

$$\sum Q = f(x,\ Q) \tag{3}$$

It will be assumed that discounted profit per barrel, e, and dry hole cost per unsuccessful well, both as absolute values, are independent in that they are not directly derived from the number of wells drilled, etc. Therefore:

$$P = \left[\Sigma Q = f(x,\ Q)\right]\ e - \left\{ n - \left[x = f(n,\ p)\right]\right\}\ C_u \tag{4}$$

In Equation 4, <u>which will be used as the profitability model in this thesis</u>, it can be seen that the discounted profit for the well drilling venture is equal to the size of reserves

discovered in the venture (which is a function of the number of successful wells and the reserve discovered by each successful well) multiplied by the discounted profit per barrel for oil which will be produced from the reserves, less the number or unsuccessful wells (a function of the number of wells drilled and the probability of successes) multiplied by the cost of the unsuccessful well, or dry hole cost.

In terms of best estimates, the best estimate of discounted profit for the venture, E(P), can be written,

$$E(P) = E(x) \left[E(Q)\ E(e) \right] - \left[n - E(x) \right] C_u \tag{5}$$

The relationship expressed in Equation 5, has been used by many to estimate the profitability of a venture directly as follows:

Based on all available information, "best guesses" are made of the probability of success, which, in this case, is the chance that at least one oil field will be found, the magnitude of the reserves which would be expected by a successful well, the profit per barrel for oil produced and the dry hole cost. It should be emphasized that normally only one value for each parameter is provided, that is <u>the</u> best estimate (or guess). These best estimates are then combined with the number of successes determined by

$$x = np \tag{6}$$

That is, if we had an 18 well program and the probability of success was 1 in 9 then we could expect 2 successes. If we multiply the 2 successes by the best guess of reserve size per

well (say 5 million barrels) we would have a total reserve of 10 million barrels. If we estimate the most likely discounted profit per barrel to be $.80 then we would expect 10 million bbls x $.80/bbl or $8 million. From this we would have to deduct dry hole costs which would be (18-2) or 16 wells at, say, $600,000 per well, or $9.6 million. The best estimate of discounted profit for the venture based on this technique is therefore - $1.6 million. In all probability the venture would not be started on the basis of this analysis.

As discussed in Chapter I, what this analysis ignores is the range of values possible for each of the parameters. For example the best estimate for reserve size was 5 million barrels per success. That may be the best estimate, but what if the reserves actually exceed this best estimate by 25%? The best estimate of discounted profit would then have been $0.8 million and a potentially profitable venture may have been passed up.

In this thesis the basic relationship expressed in Equation 4 will be utilized for the profitability analysis. In subsequent Chapters each of the parameters will be explored to determine their functional relationships and proper frequency distributions.

CHAPTER IV

DESCRIPTIVE MEASURES OF PROBABILITY DISTRIBUTIONS

4.1 Measures of Central Tendency (Mean, Median and Mode)

In analyzing each parameter in the profitability model as well as the resultant discounted profit, it is necessary to know not only its probability distribution but also some single measure typical of the parameter. This single measure will be used to describe the central tendency, or the general region of continuum in which the distribution lies.

The most common measure of central tendency is called the Mean or Expected Value and is designated $\bar{x}$ (or x_o) or $E(x)$. The expected value is a measure of location in the sense that it roughly locates a middle or "average" value of the variable, x [14]. The mean is the sum of a set of values divided by the number of values summed. Expressed mathematically the mean of a set of n numbers, x_1, x_2, x_3, ..., x_n is:

$$E(x) = \bar{x} = \frac{x_1 + x_2 + x_3 + x_4 \ldots + x_n}{n} \tag{1}$$

Or, where f_i is the frequency of each observation of x (i.e., x_1, x_2, x_3, etc.) designated as x_i we can write:

$$E(x) = \bar{x} = \frac{\sum_1^n f_i \; x_i}{n} \tag{2}$$

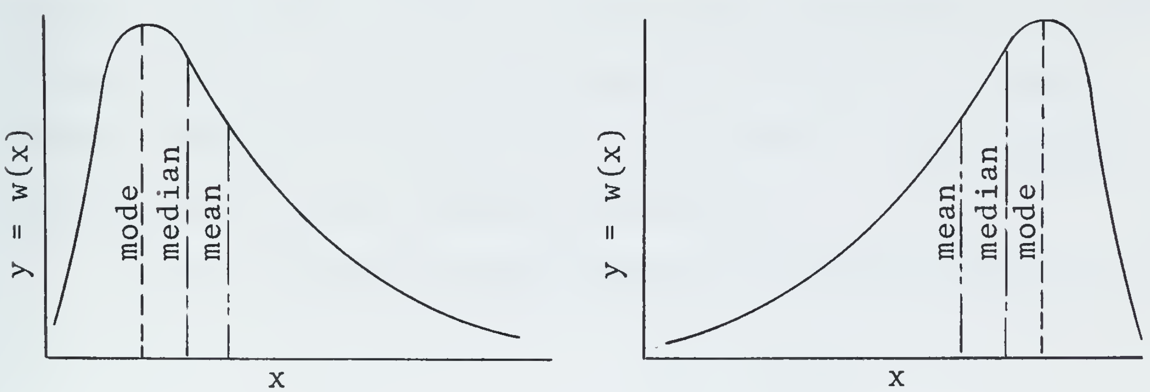
A second measure of central tendency is the Median. The median of a set of values is the middle value when the observations are arranged in either ascending or descending

order. The median is sometimes defined as the value above and below which an equal number of values occur [31]. On a cumulative probability (or frequency) plot, the median corresponds to the value opposite 50% cumulative probability (or frequency).

The third measure of central tendency is the <u>mode</u>. The mode is defined as the most frequently appearing value. It is also referred to as the "most likely value". It should be noted that a distribution does not have to have a mode (i.e. the uniform distribution). Conversely, a distribution may have more than one mode (hence the designated "bimodal" for a distribution with two modes).

Why is there a difference between these three measures of central tendency? Lawrence Mann[32] describes this difference as follows:

> This difference is due to the fact that the distribution is somewhat skewed. <u>Skewness</u> is the degree to which a distribution departs from symmetry. If a distribution leans to the right then it is said to be skewed to the left (negative skewness); if it leans to the left, then it is said to be skewed to the right (positive skewness). See Figure 4-1.



Skewed right (positive skew)   Skewed left (negative skew)

Figure 4-1 Illustration of Skewed Distribution

In this thesis we will analyze the parameters of the profitability model in terms of their mean, median and mode, as appropriate. In addition, the expected value and median for each case, or parameter set, will be measured. The expected value (mean) will be considered the best estimate of discounted profit for each parameter set and will be used, along with the median, to compare the various cases in the sensitivity studies which follow.

4.2 Measures of Variability (Variance and Standard Deviation)

The expected value of a random variable is an "average" value. However, this measure provides no information about the variability of the values of the variable. A measure of this variability, or the "spread" or "dispersion" of the values of the random variable, adds another dimension to the description of the random variable.

The most useful quantification of uncertainty associated with a future event is the standard deviation, σ. It measures the extent of the spread between individual possible outcomes of the event and the expected value of the event. Another measure of variability is provided by the variance, found by squaring the standard deviation.

For a discrete random variable, x, the variance is:

$$\sigma_x^2 = E\ (x-\bar{x})^2 = \frac{\sum_1^n f_i (x_i - \bar{x})^2}{n-1} \qquad (3)$$

where $\bar{x}$ = the mean value of x

x_i = any value of x

n = number of values of x

f_i = frequency of x_i

The denominator in Equation 2 is replaced by n-1 when the statistics are carried out on a sample size n taken from a large population - when trying to draw conclusions concerning the large population. Division by n-1 in Equation 3 makes the sample variance, ${s_x}^2$, an "unbiased" estimate of the population variance ${\sigma_x}^2$.

If x is a continuous variable, the variance is:

$$ {\sigma_x}^2 = E \quad (x-\bar{x})^2 = \int_{-\infty}^{\infty} (x_i-\bar{x})^2 \; g(x) \, dx \tag{4} $$

Evaluating the usefulness of variance as a measure of dispersion, Samuel Goldberg [15] remarks:

> One difficulty with the variance is that it does not measure dispersion in the same units as the values of x. Thus, if x has dollar values, then E(x) is a certain number of dollars, but since the variance is the mean <u>square</u> deviation, Var (x) is measured in dollars <u>squared</u>. It is in order to have a measure of dispersion in the same units as the values of x that we define the standard deviation as the square root of the variance.

Hence, variance can be adjusted for this disadvantage by taking its square root, thus defining the standard deviation, σ. This adjustment makes the standard deviation directly comparable with values of the random variable. The standard deviation takes into account, as does the expected value, not only the extreme values but all intermediate values, logically relating

uncertainty to the expected value [45].

In this thesis the standard deviation for the Base and all other parameter sets is computed by first calculating the variance using a relationship derived from Equation 3 and then taking its square root as follows:

$$\sigma_x = \sqrt{\sigma_x^2} = \sqrt{\frac{\Sigma f_i x_i^2 - n\bar{x}^2}{n - 1}} \tag{5}$$

4.3 Coefficient of Variation

A knowledge of the expected value and standard deviation gives much information about a random variable, even though one does not have any knowledge of the individual values of a random variable. However, standard deviation is a measure of absolute uncertainty, that is, it measures the actual amount of variation present in the random variable; and it is dependent on the scale of measurement. There is a need for additional important information to facilitate comparison of variation of several random variables [44].

The measure which relates uncertainty to the expected value of the random variable is the coefficient of variation, cv, the formula for which is as follows:

$$cv = \frac{\sigma_x}{\bar{x}} \tag{6}$$

where cv denotes the coefficient of variation. Use of this measure of relative uncertainty makes the variability figures directly comparable. It describes the degree of uncertainty

or relative risk of the individual expectations about the expected value of the random variable [38].

Weston and Brigham [47] describe the use of the coefficient of variation in the following passage:

> ... Use the coefficient of variation to compare the riskness of alternative investments whenever the standard deviation would be misleading. In general, the standard deviation is appropriate when percentage rates of return are considered, while the coefficient of variation is appropriate if dollar returns are considered. The reason for this distinction is that rates of return are already "standardized" for size of investment, but some form of standardization is necessary when dollar returns are concerned.

In the sensitivity studies which follow, the risk involved with each Case, or parameter set, will be measured in terms of its absolute amount (standard deviation) and in terms of its relative value (coefficient of variation). These values will then be related to the standard deviation and coefficient of variation of the Base set of parameters indicating the change in risk associated with each parameter change.

CHAPTER V

PROBABILITY OF SUCCESS

5.1 General

In assigning the probability of occurrances of future events one normally starts by examining past experiences with similar events.

The petroleum industry has been very methodical in recording drilling statistics for years. These statistics will provide the decision maker with the necessary data to compute probability of success, or, as they are called in the industry the national success ratios. These national success ratios are helpful but may be misleading. They include, for example, all wells that produced any amount of oil or gas, regardless of whether the amount was small or large, wells drilled in different formations, shallow and deep wells, and wells drilled with various kinds of skills.

While the national success ratios provide one set of past experiences they may be of little help in assuming probabilities for a particular venture in a specific location. For that particular venture local experiences rather than national experience might be more helpful. But even with local experience there are still problems in making the statistics selective so as to have relevance to the venture at hand. For instance, should all wells in all geological provinces be included? Or only those that are producing from the target horizon being sought in the venture under consideration?

These are difficult questions to answer and may be complicated by the fact that if we wanted to exclude all wells not producing from a specific geological province a breakdown of data permitting this exclusion might not be available.

It should be noted that while local and national ratios of successes do provide guides, the assignment of probability of success for a particular venture is largely subjective, and in some cases, intuitive. But, no matter how the decision is arrived at, a probability of success for the venture must be stated if a profitability analysis is to be made. Before examining how the probability of success will be used in the profitability analysis, let us examine some concepts associated with probabilities as they relate to business decisions.

For a long time only interpretation of probabilities has been the classical long-run, relative frequency arguement, i.e., repetition of the event over and over under identical conditions. For the majority of business problems, therefore, this concept has been almost useless as a practical guide for action. Most business decisions concern singular, or only occasional, events that may never be repeated - particularly under identical conditions. Therefore, choosing an act because it offers the best "long-run average" fits only a limited number of business decisions. Yet the "long-run" philosophy persists. During his research, Grayson [18] reported the following comments by operators:

> If you drill enough holes, the law of averages will take care of you. If you keep playing the averages the percentages will hit sooner of later.

When he questioned them as to what they meant by the words "sooner or later", "average", etc. most operators could not be specific.

If we say that the probability of success is 1 in 7 does this mean that if we drill 7 wells we will have one success? The answer is no! It means that if an event,such as drilling a well, is repeated over and over, the frequency of success will tend to approach the 1 in 7 success ratio. It is therefore important to remember that the number of successes is dependent on the number of wells drilled as well as the probability of success. If we consider the 1 in 7 ratio an average success ratio then, as Grayson writes [20]

> ... an operator who can drill only a small number of wells may expect large variations from the "average". The implications for a man with limited funds are clear. If he had unlimited funds, or even a very large amount of funds, he might be able to survive these variations - early losses or long "runs" of failures - while waiting for the "average to come in". But with limited funds, an operator may be ruined before the "average" is realized. It would be similar in example to the man who drowned while crossing a stream with an "average" depth of two feet.

5.2 The Binomial Distribution

An event that has a certain probability of occurrence, p, in a definite number of trials, n, is said to be binomially distributed.

When we drill an exploratory well for oil, one of two outcomes is possible. We can have a successful well, which as we will see in the next section is defined as a well which discovers a field of 1 million barrels or greater, or we can have an unsuccessful well, or a'dry hole'. If we let p equal the probability of success than 1-p, or q , is the probability of a failure. When two exploratory wells are drilled three things can happen: both can be successful, one can be successful and one dry, or both can be dry holes. The chance that both are successful is pxp or p^2, that one is successful and 1 dry is p x q + p x q or 2pq and that both are dry is q x q or q^2. Using this same logic, if three exploratory wells are drilled we can have 3 successes (p^3), 2 successes and one dry hole ($3p^2q$), 1 success and 2 dry holes ($3pq^2$) or 3 dry holes (q^3). Writing these terms in equation format we have:

$$p^3 + 3p^2q + 3pq^2 + q^3 = (p+q)^3 \qquad (1)$$

Thus, if n trials are made each of which results in either "success" or "failure", if the probability of success on each trial is p and of failure on each trial is q=1-p, and if the trials are independent, then the probabilities of all possible

results of such a trial are given by the various terms of the binomial expansion.

$$(p+q)^n = p^n + np^{n-1}q + \frac{n(n-1)}{2}p^{n-2}q^2 \;..\; \frac{n!}{x!(n-x)!}p^x q^{n-x} \;..\; +q^n, \quad (2)$$

Where the fourth term in Equation 2 is the general term and describes the probability of x successful exploratory wells in n independent trials. Hence, the binomial distribution:

$$w(x) = \frac{n!}{x!(n-x)!}\, p^x q^{n-x} \tag{3}$$

The term $\frac{n!}{x!(n-x)!}$ is often written as ${}_nC_x$ (the number of combinations). Equation 3 can therefore be expressed as:

$$w(x) = {}_nC_x\, p^x q^{n-x} \tag{4}$$

Again it is emphasized that Equation 3 provides the probability of x successes in n trials only if the probability of success, p, is constant from trial to trial and the trials are independent.

To illustrate the use of the binomial distribution in predicting successful events let us examine the following example.

Suppose we are planning an exploratory well drilling program of 20 wells and we estimate the probability of success, p, to be 1 in 7 or say 15%. Then we can also say that the probability of failure, q is 1-p, or 85%. Using Equation 4, we can write the probability of having x success in 20 trials as:

$$w(x) = {}_{20}C_x(.15)^x(.85)^{20-x} \tag{5}$$

To determine the probability of 0 successes we would set x=0, and,

$$w(0) = {}_{20}C_0(.15)^0(.85)^{20-0}$$

$$w(0) = \frac{20!}{0!\ (20)!}(.15)^0(.85)^{20}$$

$$w(0) = (.85)^{20}$$

$$w(0) = .0388 \quad \text{or} \quad 3.88\%$$

Likewise, the probability of 1 success (x=1) can be written:

$$w(1) = \frac{20!}{1!\ (20-1)!}\ (.15)^1(.85)^{19}$$

$$w(1) = \frac{2.4329 \times 10^{18}}{1.2165 \times 10^{17}}\ (.15)(.85)^{19}$$

$$w(1) = 20(.15)(.0456)$$

$$w(1) = .1368 \quad \text{or} \quad 13.68$$

A similar analysis could be performed for each success until a total of 20 were calculated. The terms could however, be read directly from tables [43]. For this example we would have the following probability for each possible outcome:

TABLE 5-1

PROBABILITY OF SUCCESS - BINOMIAL DISTRIBUTION

Number of Successes (x)	w(x) in percent
0	3.88
1	13.68
2	22.93
3	24.28
4	18.21
5	10.28
6	4.54
7	1.60
8	0.46
9	0.11
10	0.03
11	0.00
.	.
.	.
.	.
20	0.00

5.3 Simulation of the Binomial Distribution

In the Monte Carlo studies to follow we will want to simulate the binomial distribution in order to determine the number of successful wells for each drilling venture. For values of p near 50% and for large n it can be shown that the Binomial distribution approaches the Normal distribution [12]. The mean value, x_o, and standard deviation, σ, are given by:

$$x_o = np \tag{6}$$

$$\sigma = \sqrt{npq} \tag{7}$$

If x_o corresponding to a cumulative frequency, W(x) = 50%, which is a shorthand writing for $W(x \leqq x_i) = 50\%$, and $x_o + \sigma$ corresponding to a cumulative frequency, W(x) = 16% are plotted on normal probability paper and a straight line drawn between them, the probabilities associated with all other values of x could be read directly from the graph. Alternatively, the Normal distribution function

$$w(x) = \frac{1}{\sigma_x \sqrt{2\pi}} \exp \left\{ - \frac{(x-x_o)^2}{2\sigma_x{}^2} \right\} \tag{8}$$

could be solved analytically to obtain the value of w(x) for each success, x.

If the value of p varies substantially from 50% and if the value of n is relatively small can the normal distribution be used to simulate the binomial distribution? To answer this question we will continue the analysis of the 20 well drilling venture that had a probability of success of 15%. From Table 5-1 a cumulative probability, W(x), can be calculated such that $W(x \leqq x_i)$ is the chance that the number of successes will be x_i or smaller.

TABLE 5-2

CUMULATIVE PROBABILITY OF SUCCESS - BINOMIAL DISTRIBUTION

Number of Successes (x_i)	$w(x_i)$ in Percent	$(W(x \leqq x_i)$in Percent
0	3.88	3.88
1	13.68	17.56
2	22.93	40.49
3	24.28	64.77
4	18.21	92.98
5	10.28	93.26
6	4.54	97.80
7	1.60	99.40
8	0.46	99.86
9	0.11	99.97
10	0.03	100.00
11	0.00	100.00
.	.	.
.	.	.
.	.	.
20	0.00	100.00

The values of $(Wx \leqq x_i)$ corresponding to values of x on Table 5-2 were plotted on normal probability paper (Figure 5-1) as circular points. If these values, in fact approached a normal distribution then these points would fall along a straight line passing through x_o at $(Wx \leqq x_i) = 50\%$ and $x_o + \sigma$ at $W(x \leqq x_i) = 84\%$.

From Equation (6)

$$x_o = np$$

$$x_o = 20(.15)$$

$$x_o = 3$$

From Equation (7)

$$\sigma = \sqrt{npq}$$

$$\sigma = \sqrt{(20)(.15)(.85)}$$

$$\sigma = \sqrt{2.55}$$

$$\sigma = 1.60$$

Therefore, $x_o + \sigma = 4.60$

A straight line is drawn through these two points on Figure 5-1.

It will be noted from Figure 5-1 that the binomial distribution points lie off the normal distribution line. It can therefore be seen that for small values of p (in the range of 15%) and for relatively small values of n, the use of the normal distribution would not be suitable to simulate the binomial distribution for the entire range of x_i.

With this in mind the binomial distribution for this study can be simulated indirectly by assigning, for each value of x, a series of consecutive numbers equal to the values of w(x) listed on Table 5-1.

TABLE 5-3

SIMULATION OF BINOMIAL DISTRIBUTION - RANDOM NUMBERS

Number of Successes (x)	w(x) x 1000	Random Number Ranges
0	0388	0000 - 0387
1	1368	0388 - 1755
2	2293	1756 - 4048
3	2428	4049 - 6476
4	1821	6477 - 8297
5	1028	8298 - 9325
6	0454	9326 - 9779
7	0160	9780 - 9939
8	0046	9940 - 9985
9	0011	9986 - 9996
10	0003	9997 - 9999

If we now generate a four digit random number from some random number generator this number can be equated to a specific value of x. See Table 5-3. As an example, if a random number of 9395 were generated this would be equivalent to 6 successes. Likewise, if a number 0035 were generated this would equate to 0 successes, or all dry holes. With enough trials the binomial distribution can be simulated in this manner.

5.4 Coefficient of Variation

From Equations 6 and 7 the coefficient of variation can be derived as follows:

$$CV = \frac{\sigma}{x_o} = \frac{\sqrt{npq}}{np} \tag{9}$$

Since q = (1-p), Equation 9 can be written:

$$CV = \frac{\sqrt{np(1-p)}}{np}$$

From which we obtain:

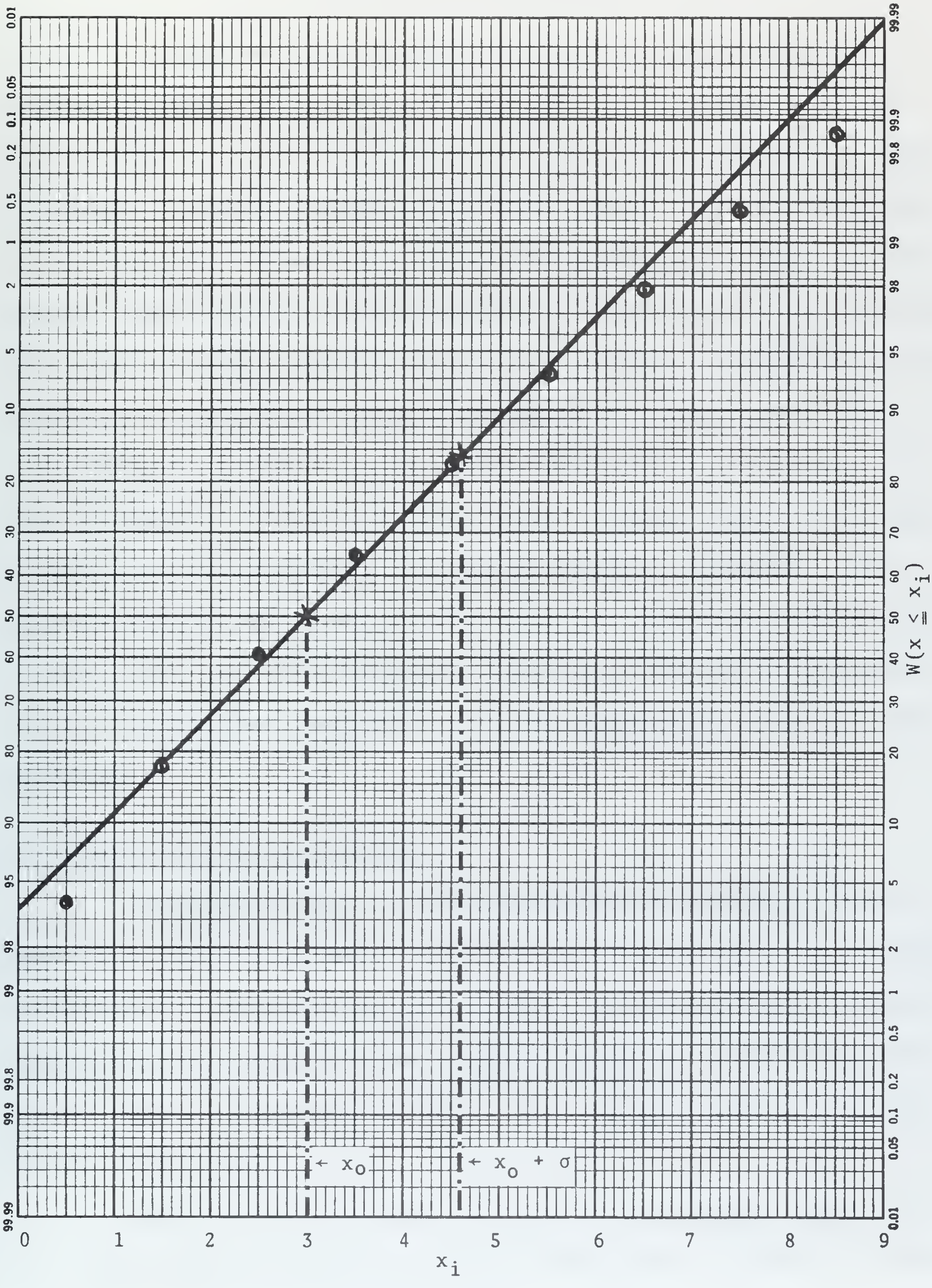
$$CV = \frac{1}{\sqrt{n}} \sqrt{\frac{1-p}{p}} \tag{10}$$

Using the data for number of wells drilled and probability of success previously noted, it is possible to calculate the Coefficient of Variation using Equation 10 as follows:

$$CV = \frac{1}{\sqrt{20}} \sqrt{\frac{1-.15}{.15}} = \frac{1}{\sqrt{20}} (\sqrt{5.60})$$

$$\underline{CV = 0.53} \tag{11}$$

Figure 5-1
The Normal & Binomial Distributions



CHAPTER VI

OIL RESERVOIR SIZE

It goes without saying that the economic success or failure of a drilling venture may well be determined by the size of the field discovered as a result of a successful well. Of course the size of the field is unknown prior to drilling. It is therefore essential, if we are going to attempt to estabilsh the relative profitability of the venture, to determine two items. First, whether there is a general mathematical expression which can be used to characterize the probability distribution of field sizes in the geological province. In other words, do the field sizes vary in such a way that their probability of occurrence can be predicted? Secondly, if a general field size distribution function can be established as a model, what are the boundary conditions we should establish for our particular venture? Each of these questions will now be explored.

6.1 Field Size Distribution

At first glance it would seem quite unlikely that a general mathematical model of field size distributions could be established - considering the complex process of oil reservoir formation, or, if by chance a model could be established for one geological province it would have general application in another province.

Some of the earliest work in this area was reported by J.J. Arps and T.G. Roberts [3] in 1958 after a study of reserve

data from the Denver-Julesburg Basin. In their study, a 5.7 million acre sample was chosen on the east flank of the Basin (located in Nebraska, Wyoming and Colorado) and the reserve estimates of all 338 fields found to that date in the Lower Cretaceous sands were investigated. Arps and Roberts arranged the fields in nine groups according to their areal extent and estimated the ultimate primary recovery from each group. The limits of these groups were chosen in such a way that each group represented fields about twice as large as the preceeding one. The nine groups thus arranged are represented on Table 6-1.

TABLE 6-1

OIL RECOVERY DATA ON DENVER-JULESBURG BASIN FIELDS

Group	Range of Productive Acres	Number of Fields	Estimated Average Ultimate Recovery (M bbls)
1	28	56	10.1
2	28 - 57	77	53.0
3	57 - 113	41	165.0
4	113 - 226	70	295.7
5	226 - 453	40	957.6
6	453 - 905	33	1,876.2
7	905 - 1810	19	4,481.6
8	1810 - 3620	1	11,460.0
9	3620	1	50,750.0

When a frequency - density distribution of the data presented in Table 6-1 was plotted versus Average Ultimate Recovery on logarithmic-probability paper a straight line can be best fit through the resulting points. (See Figure 6-1). This demonstrates that the field sizes in the Denver-Julesburg Basin are Lognormally distributed.

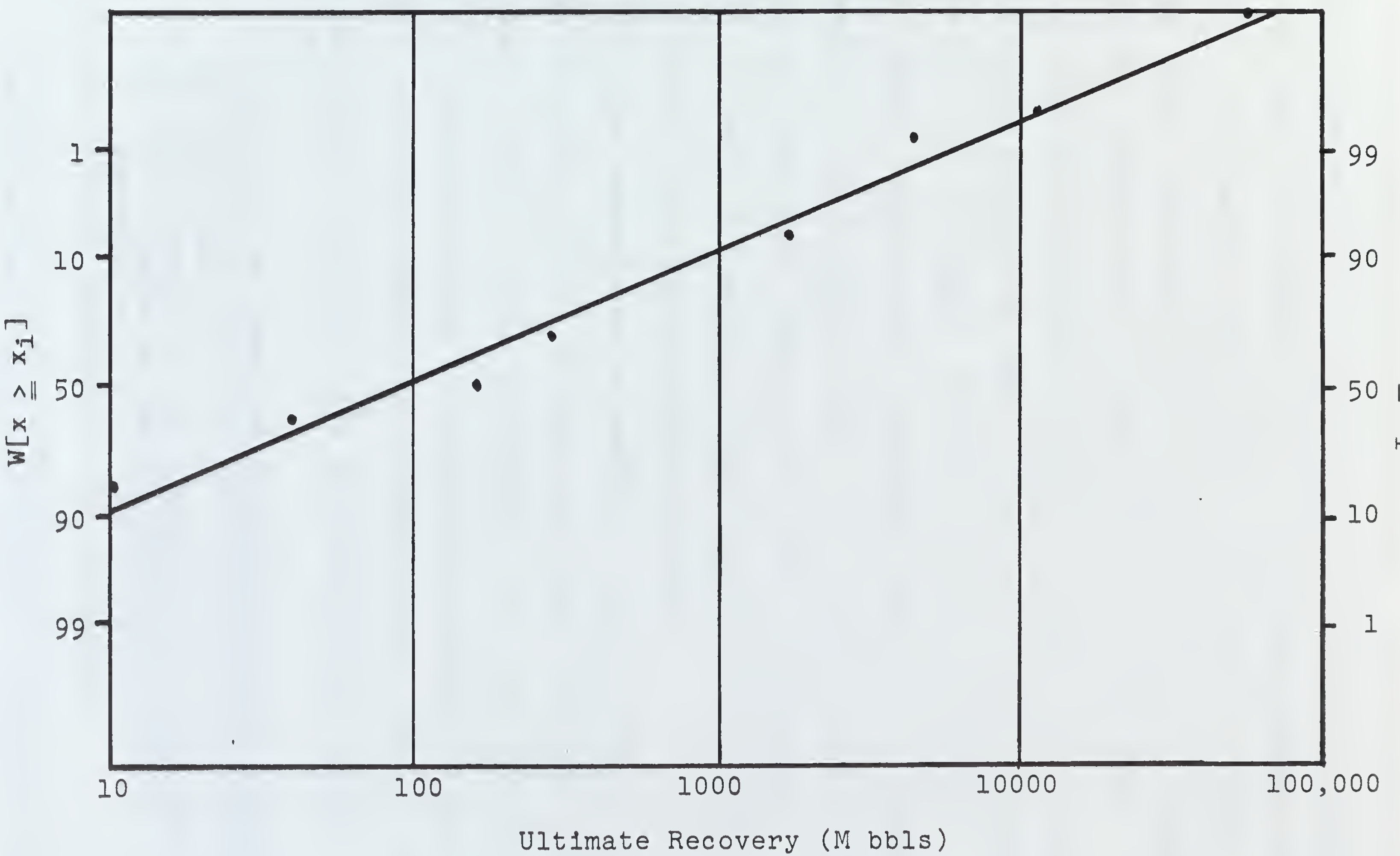


Figure 6-1 Ultimate Recovery - Denver-Julesberg Basin

While a Lognormal distribution best fits the data in the Denver-Julesburg Basin there is still no reason to generalize and say that all reserves follow a Lognormal frequency distribution. If, however, the Lognormal distribution remains invariant under a wide variation in the definition of reserves and geological regions then a generalization can be made.

To test this invariancy Gordon M. Kaufman [27], in an extensive study reported in 1962, accumulated data from several different sources including: The Oil and Gas Conservation Board of Alberta Report to the Lieutenant-Governor-in-Council as summarized in The Financial Post; Oil and Gas Journal statistics on two states for 1946 and for 1960; an individual company's estimate of reserves in four basins; and the Arps - Roberts article on the Denver-Julesburg Basin.

The data accumulated by Kaufman, covering fields in four coastal basins, fields in North and South Louisiana (both in 1946 and in 1959), fields in Oklahoma, etc. were plotted on sixteen (16) separate graphs on logarithmic-probability paper. Kaufman, in analyzing the results of these plots, writes [28]

> The graphs displaying this data tentatively suggest that:
>
> 1. Within a given area the functional form of empirical histograms of reported field sizes does not change
>   (a) over time,
>   (b) with changes in the definition of "reserves",
>   (c) with changes in the minimum size of field reported, or
>   (d) from one geographic area to another.
>
> 2. The functional form is the same for a well-defined geological basin such as the Denver-Julesburg as for an arbitary geographic area such as a state.

> Although each of /the sixteen (16) graphs/ is a plot of data from fields of varying geological ages and of varying length of production lives, the invariance in the functional forms is striking enough to warrant the conjecture that added data blocked on these two factors would not alter the conclusions stated above.

A visual observation of these graphs reveals that most points generally fall along a straight line indicating a Lognormal frequency distribution of field size.

In another analysis of oil field size distributions, Folkert Brons reports the result of a study of the predicted ultimate recovery of 75 Southern Louisiana Miocene Fields [10]. As shown on Figure 6-2, when the data is plotted on logarithmic-probability paper the points fall along a straight line indicating a Lognormal distribution function for these fields.

On the basis of the evidence presented in the studies cited above, of reservoirs with varying reserve sizes, geologic origin and physical characteristics, it will be assumed for further analysis that a Lognormal distribution function can serve as a general mathematical model for reservoir size distributions.

6.2 The Lognormal Distribution

The lognormal distribution can be defined as the distribution of a random veriable whose distribution obeys a normal distribution function. Thus, if x is lognormally distributed then y = ln x is normally distributed and,

$$w(y) = \frac{1}{\sigma_y \sqrt{2\pi}} \exp\left\{\frac{-(y-y_0)^2}{2\sigma_y^{\,2}}\right\} \qquad (1)$$

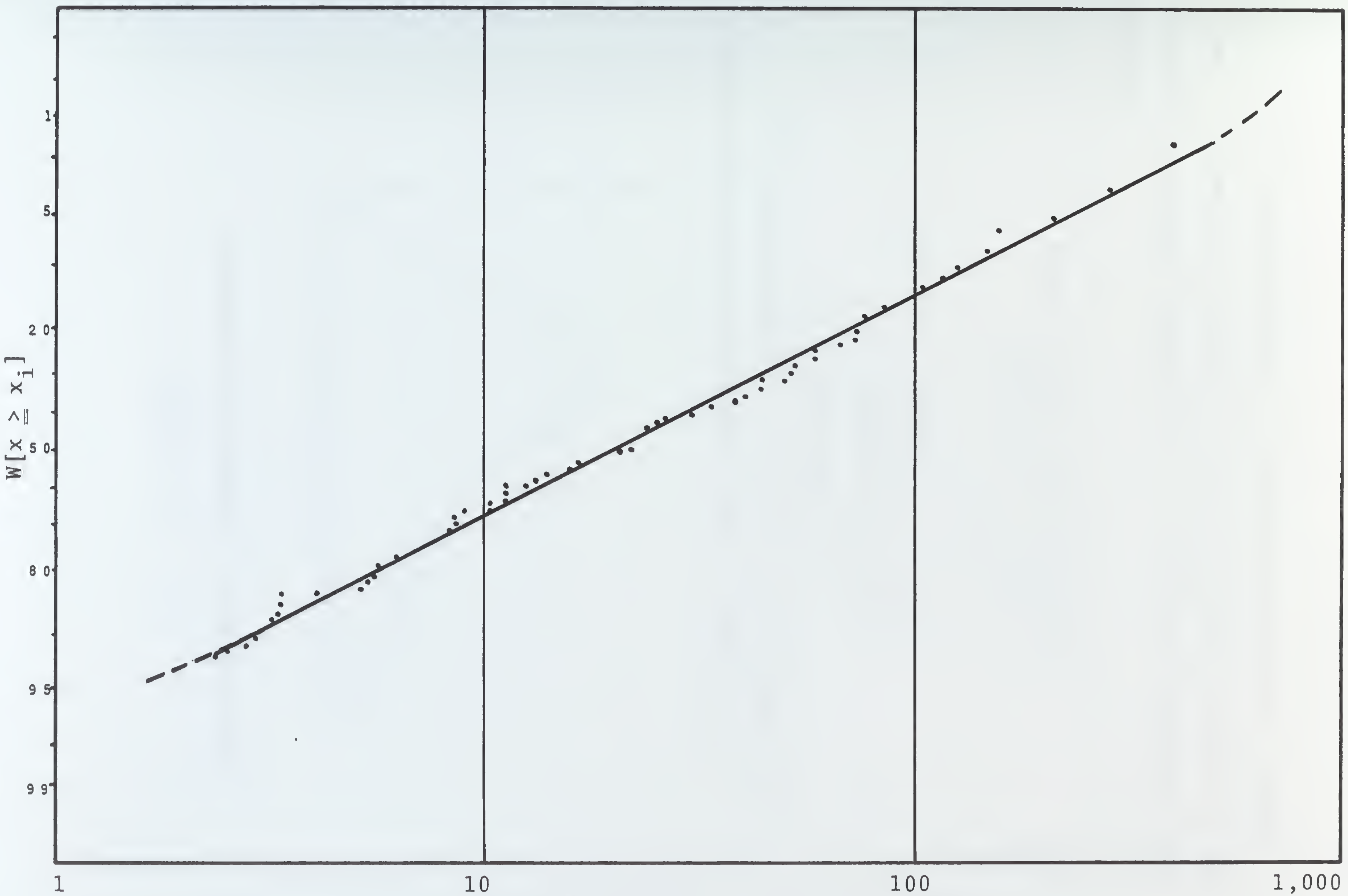


Figure 6-2 Ultimate Recovery - Southern Louisiana Miocene Fields

If the distribution is desired in terms of x rather than y a change in variables is performed which results in the probability density function for y expressed in terms of x as follows:

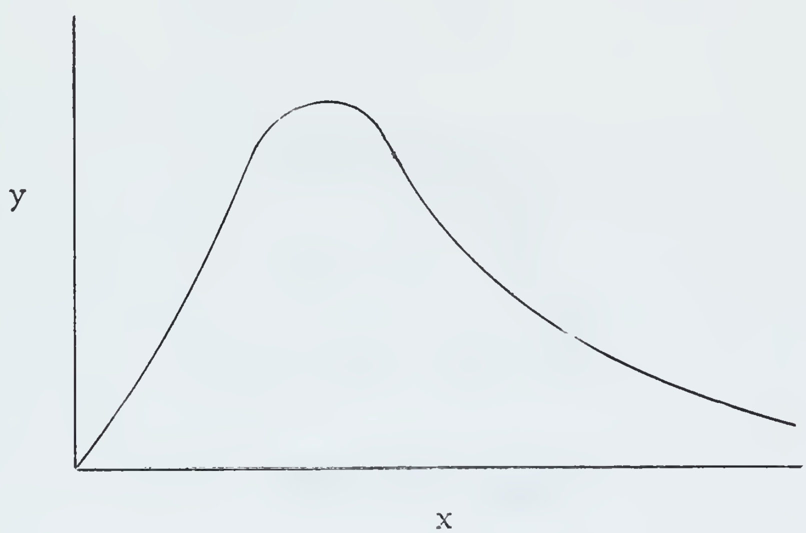
$$w(x) = \frac{1}{\sigma_y \sqrt{2\pi}} \frac{\exp\left\{\dfrac{-(\ln x - y_o)^2}{2\sigma_y^2}\right\}}{x} \qquad (2)$$

where: x = independent variable

σ_y = standard deviation

y_o = mean value of y

The equation gives a lognormal distribution curve, with its characteristic positive skew, as shown below.



Lognormal Distribution - Positive Skew

Figure 6-3

Since y is normally distributed:

$$y_m(\text{median} = y_o(\text{mean}) = y(\text{mode}), \qquad (3)$$

but since $x = e^y$, x is not normally distributed and the values of x_m(median), x_o(mean) and x(mode) must be calculated separately [9]:

<u>The Median, x_m:</u>

$$y = \ln x$$

$$x_m = e^{y_m} \qquad (4)$$

since $y_m = y_o$

$$x_m = e^{y_o} \qquad (4A)$$

and, $\ln x_m = y_o$ (4B)

<u>The Mean, x_o:</u>

$$x_o = e^{y_o + \frac{\sigma y^2}{2}} \qquad (5)$$

or, $\ln x_o = y_o + \frac{\sigma y^2}{2}$ (5A)

<u>The Mode, x_{mode}:</u>

$$x_{mode} = e^{y_o - \sigma y^2} \qquad (6)$$

or, $\ln x_{mode} = y_o - \sigma_y{}^2$ (6A)

Further, the Variance, σ_x^2 is given by the following expression:

$$\sigma_x^2 = \exp\{ 2y_o + \sigma_y^2 \} (e^{\sigma_y^2}-1) \tag{7}$$

Inserting Equation 5 in Equation 7 we obtain:

$$\sigma_x^2 = x_o^2 (e^{\sigma_y^2}-1) \tag{8}$$

From which the Coefficient of Variation, CV, may be expressed as:

$$CV = \frac{\sigma_x}{x_o} = e^{\sigma_y^2}-1 \tag{9}$$

For small values of σ_y we can approximate Equation 9 as:

$$CV = \sigma_y \tag{10}$$

6.3 Boundry Conditions

If we assume that the field (or reservoir) sizes in any area are lognormally distributed and will fall along a straight line when plotted on logarithmic-probability paper then the next problem is one of specifing the position of the line which would represent the reservoir size distribution in the specific location where the drilling venture is to be conducted.

This line could be drawn if someone could predict the median value of the reserve size to be discovered by a (yet undrilled) successful well (i.e. the reserve size corresponding to a 50% probability) and the standard deviation of the distribution representing the logarithm of the field sizes.

Then, by inserting these values in Equations (4B), (5A) and (6A) previously noted for median, mean and mode a straight line could be drawn on log-probability paper. The problem is, however, that the operator (or his agent) would probably not be able to specify either the median or the standard deviation. What he most likely would be able to specify is his estimate of the highest and lowest values of reserves he would expect with, say, 90% certainty. He would probably base these estimates on either his own experience or the experience of others who have drilled in the area. As an example, the operator may not be able to specify an exact reserve size, although he might say that he would expect the reserves to run from 5 million to 150 million barrels with 90% confidence. If these two points were plotted on log-probability paper - with the 5 million barrel figure plotted at the 5% probability point and the 150 million barrel figure plotted at the 95% probability point a straight line could be drawn between them which would represent the Lognormal distribution of all reserves in the area.

With the line plotted, the median value of Q (or x_m in Equation 4B), corresponding to a 50% probability, and the value of Q median plus one standard deviation, corresponding to an 84% probability, can be read directly. In addition, to use this model in our determination of the relative profitability of the venture using the Monte Carlo technique, it is essential that we be able to determine a reservoir size and corresponding value of probability (that the reservoir size will not be exceeded) for each point along the line.

To illustrate this procedure, as well as to generate data for use in the Monte Carlo studies conducted as part of this thesis, let us assume we are going to drill for oil on-shore in the Southern Louisiana area. By going to the appropriate issue of the Oil and Gas Journal [48] we would find a listing of the sizes of all fields in the Southern Louisiana on-shore area. These fields could then be arranged in ascending order of size (Q) and a relative frequency density, w, and cumulative frequency, $W(Q \leqq Q_i)$ calculated. This tabular listing, along with the calculation of relative and cumulative frequency is shown as ACTUAL Values in Table 6-3.

Having the actual values of cumulative frequency calculated, these values can be plotted on log-probability paper with the ordinate being the cumulative frequency and the abscissa being the field size. The triangular points on Figure 6-4 represent a plot of the actual data for the Southern Louisiana on-shore area.

The operator can now use these points (or any other data he has available) as a guide in drawing a straight line on the log-probability paper beginning at some upper value of Q, corresponding to a 95% probability, and at some lower value of Q, corresponding to a 5% probability. In this example the points chosen were 79 million barrels as the upper value and 5 million barrels as the lower value. With this line drawn (Curve A, Figure 6-4) it can now be said that, for the purposes of this profitability analysis, 90% of all reserves discovered by a successful well will fall along the straight line, 5% of

them will exceed the upper value and 5% will be less than the lower value. Theoretically then, there is a 5% possibility that a reservoir could be discovered with infinite reserves on one extreme and a 5% possibility of a reservoir with a 0 reserve on the other. Practically of course, this is not reasonable since 0 reserve means no oil and an unsuccessful well. If this straight line represents the reserve size distribution for a successful well then some lower limit must be established which would represent the reserve size below which the well in question would not be considered successful, even though some oil was discovered. In this example a lower limit of 1 million barrels was chosen. Therefore, we say to be considered successful a well must discover at least a 1 million barrel reserve.

At the upper extremes it would be foolish to say that an infinite reservoir could be discovered even though this is possible by an extrapolation of Curve A, Figure 6-4, to a 100% probability point. Realizing that this is not physically possible an arbitrary upper limit should be established. In this example it was observed that in the Southern Louisiana on-shore area the maximum field size is 137 million barrels (see Table 6-3, ACTUAL). On this basis an upper limit of 140 million barrels was chosen.

6.4 The Lognormal Distribution of Field Sizes

With the boundary conditions now established and Curve A,Figure 6-4 drawn, it is now possible to proceed with the calculation of the mean, median and mode as follows:

<u>The Median, x_m:</u>

Q at 50% = <u>20.0 million barrels</u>

$$\text{let } Q_{median} = x_m$$

From Equation 4B

$$y_o = y_m = \ln x_m$$

$$y_o = \ln 20.00$$

$$y_o = 2.996$$

$$y_o + \sigma_y = \ln x \text{ (at } W \leqq x = 84\%)$$

$$y_o + \sigma_y = \ln 46.0$$

$$2.996 + \sigma_y = 3.829$$

$$\sigma_y = 0.833$$

$$\sigma_y^2 = 0.694$$

$$\frac{\sigma_y^2}{2} = 0.347$$

<u>The Mean, x_o:</u>

From Equation 5A

$$\ln x_o = y_o + \frac{\sigma_y^2}{2}$$

$$\ln x_o = 2.996 + 0.347$$

$$\ln x_o = 3.343$$

$$\underline{x_o = 28.3 \quad \text{million barrels}}$$

<u>The Mode, x_{mode}:</u>

From Equation 6A

$$\ln x_{mode} = y_o - \sigma_y{}^2$$

$$\ln x_{mode} = 2.996 - 0.694$$

$$\ln x_{mode} = 2.302$$

$$\underline{\ln x_{mode} = 10.0 \quad \text{million barrels}}$$

From Equation 9

$$CV = \frac{\sigma_x}{x_o} = \sqrt{e^{\sigma_y{}^2} - 1}$$

$$CV = \sqrt{e^{0.694} - 1}$$

$$CV = \sqrt{2.0017 - 1} = \sqrt{1.0017}$$

$$\underline{CV = 1.00}$$

If we assign random numbers to represent $W(Q \leqq Q_i)$ we can, from Curve A, Figure 6-4, manually determine the corresponding value of Q as follows:

TABLE 6-2

FIELD SIZES - RANDOM NUMBERS

Random No.	$W(Q \leqq Q_i)$ (in %)	Q ($\times 10^6$ bbl)
.	.	.
.	.	.
.	.	.
05	05	5.0
06	06	5.5
07	07	5.8
.	.	.
.	.	.
.	.	.
93	93	69.0
94	94	73.0
95	95	80.0

While the process of manually reading the graph, tabulating the values of Q corresponding to Random Variables which represent $W(Q \leqq Q_i)$ and inputing these numbers as data in the Monte Carlo computer program is possible, the process is quite laborious. To equip ourselves for an efficient simulation system a Lognormally Distributed Variable Generator can be used to generate the values of Q on a high speed digital computer.

Such a Generator was initially developed by McMillan and Gonzalez [34]. A version of the Generator, modified for use in this problem, incorporating the concepts of upper and lower limits, is included in Appendix A. Using this Generator it is necessary only to input the median Value of Q and the value of Q at $W(Q \leqq Q_i) = 84\%$, both easily obtainable from

the plot of a straight line connecting the 5% and 95% points, and the upper and lower limits, as previously discussed. With this data inputed, the Generator will provide field sizes (Q) which are lognormally distributed within the boundary conditions established for the program.

In order to test the reliability of the deviates provided by the Lognormal Generator, 1000 values of Q were generated using the program included in Appendix A and the boundary conditions previously discussed. The results of this test, along with a calculation of relative and cumulative frequency, is included on Table 6-3 under the heading of SIMULATION. For comparison purposes the values provided by the Generator were placed beside the corresponding actual values. Additionally, Curve B, Figure 6-4 was plotted with points generated by the Lognormal Generator.

By comparing Curves A and B and the actual data points on Figure 6-4 it will be seen that excellent correlation exists except in the lower values of Q - but even there the difference is not considered significant. On this basis,the Lognormally Distributed Variable Generator will be used to simulate lognormally distributed values of field size in this thesis.

Figure 6-4
Cumulative Distribution of Field Sizes
Southern Louisiana On-Shore

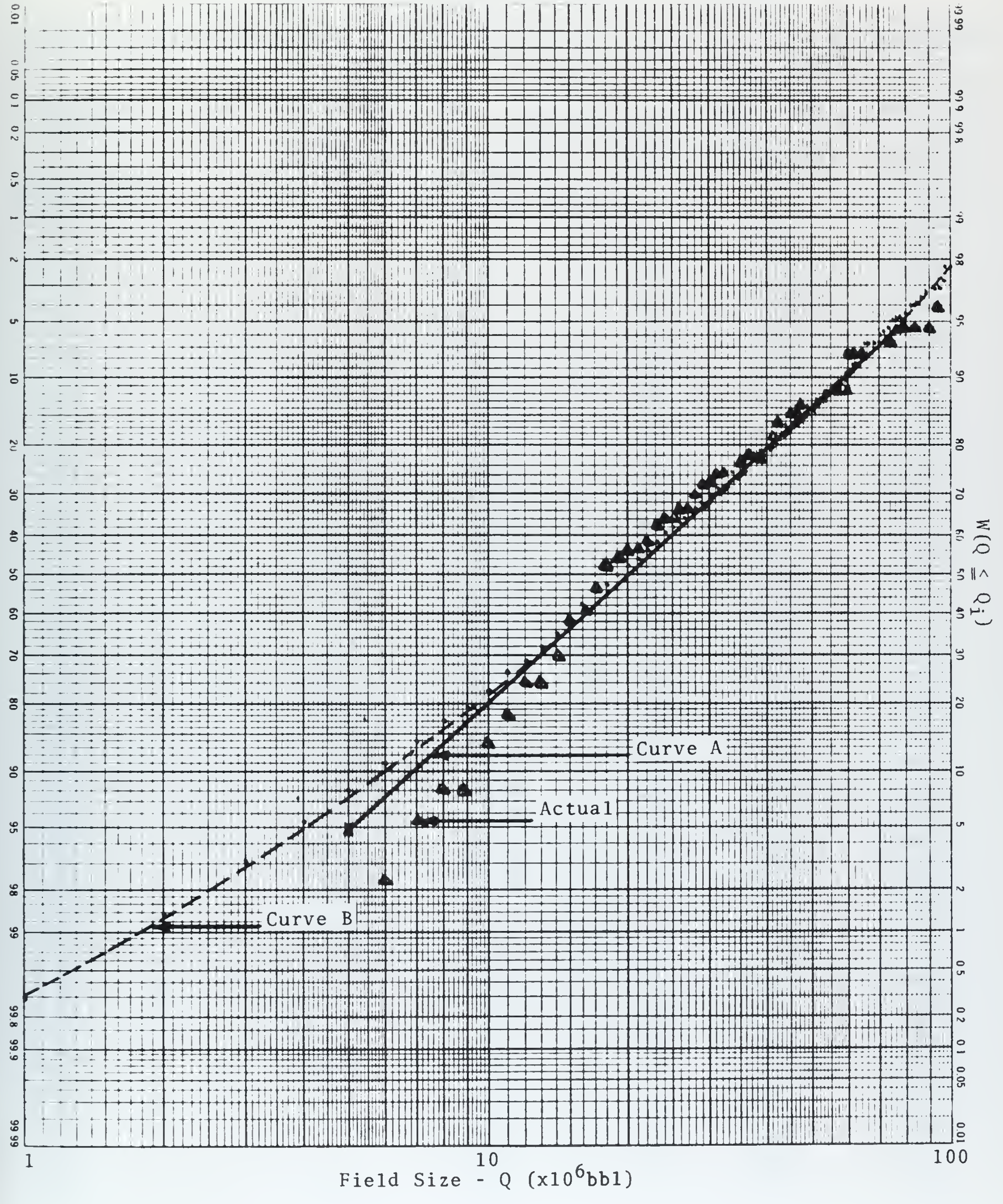


TABLE 6-3

SIZE OF FIELD - SOUTHERN LOUISIANA ON-SHORE
(1970 Estimates)

Field Size-Q ($x10^6$ bbl)	ACTUAL Freq. (f)	ACTUAL Rel.Freq. ($w=\frac{f}{n}x100$) (%)	ACTUAL Cum.Freq. ($W \leq Q$) (%)	SIMULATION Freq. (f)	SIMULATION Rel.Freq. ($w=\frac{f}{n}x100$) (%)	SIMULATION Cum.Freq. ($W \leq Q$) (%)
1				3	0.3	0.3
2				11	1.1	1.4
3				18	1.8	3.2
4				25	2.5	5.7
5				24	2.4	8.1
6	2	2.4	2.4	33	3.3	11.4
7	3	3.5	5.9	27	2.7	14.1
8	2	2.4	8.3	34	3.4	17.5
9	0		8.3	19	1.9	19.4
10	5	5.9	14.2	45	4.5	23.9
11	4	4.7	18.9	34	3.4	27.3
12	5	5.9	24.8	25	2.5	29.8
13	0		24.8	23	2.3	32.1
14	5	5.9	30.7	37	3.7	35.8
15	7	8.2	38.9	28	2.8	38.6
16	3	3.5	42.4	37	3.7	42.3
17	4	4.7	47.1	32	3.2	45.5
18	5	5.9	53.0	25	2.5	48.0
19	2	2.4	55.4	25	2.5	50.5
20	1	1.2	56.6	27	2.7	53.2
21	1	1.2	57.8	21	2.1	55.3
22	1	1.2	59.0	18	1.8	57.1
23	4	4.7	63.7	22	2.2	59.3
24	1	1.2	64.9	20	2.0	61.3
25	0		64.9	11	1.1	62.4
26	1	1.2	66.1	19	1.9	64.3
27	1	1.2	67.3	15	1.5	65.8
28	3	3.5	70.8	12	1.2	67.0
29	1	1.2	72.0	13	1.3	68.3
30	1	1.2	73.2	20	2.0	70.3
31	1	1.2	74.4	15	1.5	71.8
32			74.4	8	0.8	72.6
33			74.4	13	1.3	73.9
34			74.4	18	1.8	75.7
35	2	2.4	76.8	6	0.6	76.3
36	1	1.2	78.0	13	1.3	77.6
37			78.0	4	0.4	78.0
38			78.0	8	0.8	78.8
39			78.0	7	0.7	79.5
40	2	2.4	80.4	6	0.6	80.1

TABLE 6-3 Continued

Field Size-Q ($x10^6$bbl)	ACTUAL Freq. (f)	RelFreq. ($w=\frac{f}{n}x100$) (%)	Cum.Freq. ($W \leq Q$) (%)	SIMULATION Freq. (f)	Rel.Freq. ($w=\frac{f}{n}x100$) (%)	Cum.Freq. ($W \leq Q$) (%)
41	1	1.2	81.6	5	0.5	80.6
42	2	2.4	84.0	8	0.8	81.4
43			84.0	9	0.9	82.3
44			84.0	3	0.3	82.6
45	1	1.2	85.2	6	0.6	83.2
46			85.2	8	0.8	84.0
47	1	1.2	86.4	4	0.4	84.4
48			86.4	9	0.9	85.3
49			86.4	4	0.4	85.7
50			86.4	2	0.2	85.9
51	1	1.2	87.6	9	0.9	86.8
52			87.6	5	0.5	87.3
53	1	1.2	88.8	5	0.5	87.8
54			88.8	2	0.2	88.0
55			88.8	6	0.6	88.6
56			88.8	5	0.5	89.1
57			88.8	2	0.2	89.3
58			88.8	7	0.7	90.0
59	1	1.2	90.0	3	0.3	90.3
60	2	2.4	92.4	6	0.6	90.9
61			92.4	6	0.6	91.5
62			92.4	2	0.2	91.7
63			92.4	7	0.7	92.4
64	1	1.2	93.6	2	0.2	92.6
65			93.6	1	0.1	92.7
66			93.6	5	0.5	93.2
67			93.6	1	0.1	93.3
68			93.6	2	0.2	93.5
69			93.6	4	0.4	93.9
70			93.6	3	0.3	94.2
71			93.6	3	0.3	94.5
72			93.6	3	0.3	94.8
73			93.6			94.8
74			93.6	3	0.3	95.1
75	1	1.2	94.8	1	0.1	95.2
76			94.8	2	0.2	95.4
77			94.8	1	0.1	95.5
78			94.8	1	0.1	95.6
79			94.8	1	0.1	95.7
80			94.8			95.7
81			94.8	1	0.1	95.8
82			94.8	3	0.3	96.1

TABLE 6-3 Continued

Field Size-Q ($x10^6$bbl)	ACTUAL Freq. (f)	ACTUAL Rel.Freq. ($w=\frac{f}{n}x100$) (%)	ACTUAL Cum.Freq. ($W \leqq Q$) (%)	SIMULATION Freq. (f)	SIMULATION Rel.Freq. ($w=\frac{f}{n}x100$) (%)	SIMULATION Cum.Freq. ($W \leqq Q$) (%)
83			94.8			96.1
84			94.8	1	0.1	96.2
85			94.8	4	0.4	96.6
86			94.8			96.6
87			94.8			96.6
88			94.8	2	0.2	96.8
89			94.8			96.8
90			94.8	1	0.1	96.9
91			94.8			96.9
92			94.8			96.9
93	1	1.2	96.0	1	0.1	97.0
94			96.0	3	0.3	97.3
95	1	1.2	97.2	4	0.4	97.7
96			97.2	2	0.2	97.9
97			97.2			97.9
98			97.2			97.9
99			97.2			97.9
100			97.2			97.9
101			97.2	2	0.2	98.1
102			97.2	1	0.1	98.2
103			97.2	1	0.1	98.3
104			97.2			98.3
105			97.2	1	0.1	98.4
106			97.2	1	0.1	98.5
107			97.2			98.5
108	1	1.2	98.4			98.5
109			98.4			98.5
110	1	1.2	99.6			98.5
111			99.6			98.5
112			99.6			98.5
113			99.6	1	0.1	98.6
114			99.6	1	0.1	98.7
115			99.6			98.7
116			99.6			98.7
117			99.6	1	0.1	98.8
118			99.6			98.8
119			99.6	1	0.1	98.9
120			99.6			98.9
121			99.6			98.9
122			99.6			98.9
123			99.6	3	0.3	99.2
124			99.6			99.2
125			99.6	1	0.1	99.3

TABLE 6-3 Continued

Field Size-Q ($x10^6$bbl)	ACTUAL Freq. (f)	ACTUAL Rel.Freq. ($w=\frac{f}{n}x100$) (%)	ACTUAL Cum.Freq. ($W \leqq Q$) (%)	SIMULATION Freq. (f)	SIMULATION Rel.Freq. ($w=\frac{f}{n}x100$) (%)	SIMULATION Cum.Freq. ($W \leqq Q$) (%)
126			99.6			99.3
127			99.6			99.3
128			99.6			99.3
129			99.6			99.3
130			99.6			99.3
131			99.6			99.3
132			99.6			99.3
133			99.6			99.3
134			99.6	1	0.1	99.4
135			99.6			99.4
136			99.6			99.4
137	1	1.2	100.0			99.4
138			100.0	1	0.1	99.5
139			100.0			99.5
140			100.0	5	0.5	100.0
	85			1000		

CHAPTER VII

DISCOUNTED PROFIT PER BARREL

7.1 General

In a very real sense, the ultimate profitability of an exploratory well drilling venture, where oil is discovered, is dependent not only on the size of the new reserves but also (and perhaps more importantly) on how much profit the operator will receive for each barrel of oil produced. The case can be visualized where the operator in a successful exploratory drilling venture discovers vast potential oil reserves but the expence of production far exceeds the posted price he can receive for the oil produced. In this case no matter how large the potential reserve, the profitability of the venture is less than 0, or if produced, the operator would suffer a loss. The cases of off-shore drilling in deep water and production on the North Slope of Alaska (the latter receiving wide publicity) illustrate this notion. It is therefore essential in a profitability analysis to estimate the expected profit per barrel on reserves discovered by a successful well. In practice, though, it is rare that the operator would commit himself to "the" expected profit per barrel for the venture. He could perhaps provide his estimates of the lowest, highest and most likely values of profit per barrel basing these estimates on his experience in the area or on factors similar to those which permitted him to estimate the probability of success (see Chapter V).

7.2 Discounted Profit Per Barrel Estimate

In estimating the highest lowest and most likely values of profit per barrel the operator would first have to provide his estimate of the highest, lowest and most likely posted price for the oil. From this he would have to deduct his expenses including, royalties, severance taxes, operating costs, drilling and completion costs, etc. This would then provide his net profit per barrel for each or the three estimates. But, since reserves cannot be produced all at once, the profit per barrel can not be figured on the basis of current sales prices. Production will be spread into the future (say 10 years) and future money is not as valuable as present day money. To account for this financial fact-of-life the operator will have to convert his net profit per barrel calculated at today's prices into a discounted cash flow. In this discounting (or deferment) process a discounting factor is developed by calculating the Present Worth of a $1 annuity (i.e. constant income or expenditure) received each year for a specified length of time, n, at a specified discounted rate, i, [16]. This is given by the expression:

$$P = R \left[\frac{(1 + i)^n - 1}{1(1 + i)^n} \right] \qquad (1)$$

where: P is the Present Worth

R is the amount of the annuity (taken as $1 in this development)

$$\left[\frac{(1 + i)^n - 1}{1(1 + i)^n}\right]$$ is the uniform series Present Worth or Present Value Factor.

The Present Worth Factor can also be written as

$$\left[\frac{1 - \frac{1}{(1 + i)^n}}{1}\right]$$ or Values of the Present Worth can be obtained from Tables. If these values are then divided by the number of discounting periods, n, the deferment or discounting factor is obtained.

To illustrate, the discount factor based on a discount rate of 10% for a period of 10 years is calculated as follows:

From Equation 1,

$$P = R\left[\frac{(1 + i)^n - 1}{i(1 + i)^n}\right]$$

Let $R = \$1$

$i = 10\%$

$n = 10$ years

From Tables [17] the Present Worth Factor is 6.1444

$$P = 1(6.1444)$$

$$P = 6.1444$$

$$\text{Discount factor} = \frac{P}{n} \qquad (2)$$

$$\text{Discount factor} = \frac{6.144}{10}$$

$$\text{Discount factor} = .6144$$

Concerning this technique, Grayson [19] writes:

> Many operators, ... object to the present value concept: First, there are those who scoff at the idea of making any present value computations on exploratory ventures. There are already so many uncertainties, is their belief, that to refine the calculation any further is

> like tilting at windmills. The answer to this has been stated several times: a present value calculation is an estimate, but an educated one that explicitly recognizes time as a realistic decision consideration. To ignore the effect of time or to try to cope with it in an implict way is to incur the danger that it may be overlooked entirely or its effect miscalculated.

On the basis of the concepts discussed in this section, the highest, lowest and most likely values of profit per barrel can be computed. Table 7-1 illustrates this process and is based on the hypothetical expenses and prices for a drilling venture in the Southern Louisiana on-shore area.

The values developed in Table 7-1 will be used as the base figures for the sensitivity studies contained in subsequent Chapters.

7.3 The Triangular and Beta Distributions

It was seen in Section 7.2 that while the operator would probably be unable to provide the exact profit per barrel of oil produced he could provide his estimates of the highest profit per barrel he would expect, the lowest profit per barrel and the profit per barrel he feels most likely to occur. It is assumed of course that these figures will be discounted to reflect the time value of money. Given this data, the question then becomes, what distribution function best suits this set of parameters? If all that is known is the upper limit, the lower limit and most likely value (mode) it is possible to describe this knowledge by means of either the triangular distribution function or the beta distribution function. Of the two, the triangular function is the simplest and will be discussed first.

TABLE 7-1

DISCOUNTED PROFIT PER BARREL - SOUTHERN LOUISIANA ON-SHORE

ITEM	MOST LIKELY	HIGH	LOW
POSTED PRICE	$3.25	3.50	3.00
DEDUCTIONS:			
ROYALTY:			
@1/6	.54		.50
@1/8		.44	
SEVERANCE TAXES (see Note 1)	.24	.26	.22
OPERATING COSTS (including lift cost, labor, lease maint., etc.)	.10	.11	.09
HOME OFFICE COSTS OR OVERHEAD (@ 5%)	.16	.18	.15
GEOPHYSICAL COSTS	.01	.01	.01
DRILLING & COMPLETION (see Note 2)	.40	.40	.40
LEASE ACQUISITION (see Note 3)	.02	.02	.02
TOTAL DEDUCTIONS	-$1.82	-1.62	-2.19
NET PROFIT	1.43	1.88	.81
DISCOUNTED PROFIT (based on discounting at 10% for 10 years: discount factor = .6144)	$.88	$1.15	$.50

NOTE 1: [49] a. State and local severance taxes paid in 1968 = $233,017,000.

b. Total Petroleum liquid production in 1968 = 2,526,008 bbl/day (average).

c. Average field price of oil (1969) = $3.33

$$\frac{\$233,071,000}{2,526,008 \text{ bbl/day} \times 365} = \$.252/\text{bbl}$$

TABLE 7-1 Continued

however, at \$3.30/bbl

$$\text{Severance taxes} = \frac{.252}{3.30} = 7.35\%$$

say 7.4% of posted price

NOTE 2:[50]

a. Assume production at 150 bbl/day.

b. Estimated Cost of drilling and equipping wells in 1970 = \$196,500. Allowing for inflation & deeper wells assume cost of drilling = \$217,500/well.

c. Assume recovery of 217,500 in 10 years then recovery/year = \$21,750.

$$\frac{\$21,750}{150 \text{ bbl/day} \times 365} = \$.40/\text{bbl}$$

NOTE 3:

a. Assume Lease Costs at \$60/acre where a completion is made.

b. Assume well spacing at 40 acres; then Lease Costs \$2400/well where well is completed.

c. Assume lease on unproductive land to be equivalent to \$2400/well; then total lease costs = \$4,800/well.

d. Assume recovery in 5 years with production rates = 150/bbl/well.

$$\frac{\$4800}{5} = \$960/\text{well}$$

$$\frac{\$960}{150 \text{ bbl/day} \times 365} = .017$$

say \$.02/bbl

7.4 The Triangular Distribution Function

The triangular distribution is defined by three values of the variable; i.e. the lowest valur of x = a, the highest value of x = c and the most likely value of x = b. The probability distribution function w(x) can take three forms as shown below:

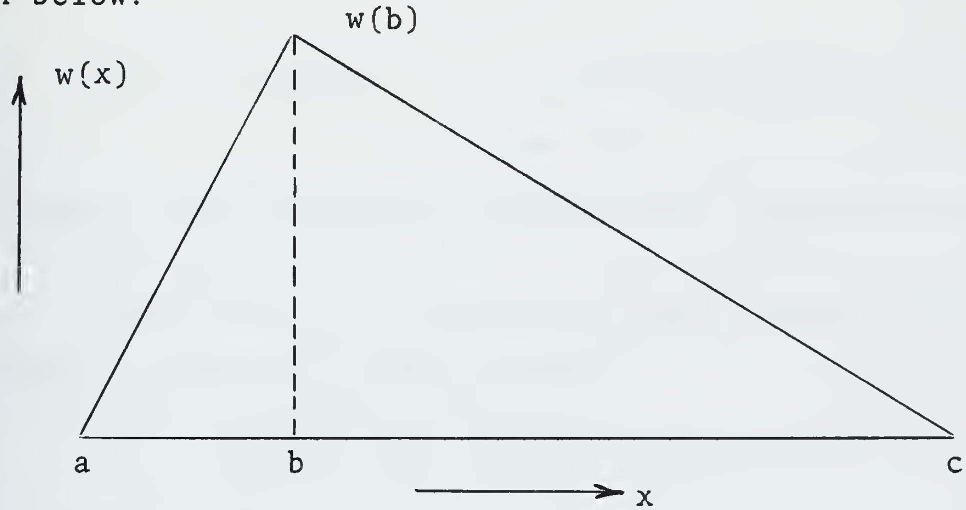


Figure 7-1A Triangular Distribution - Positive Skew

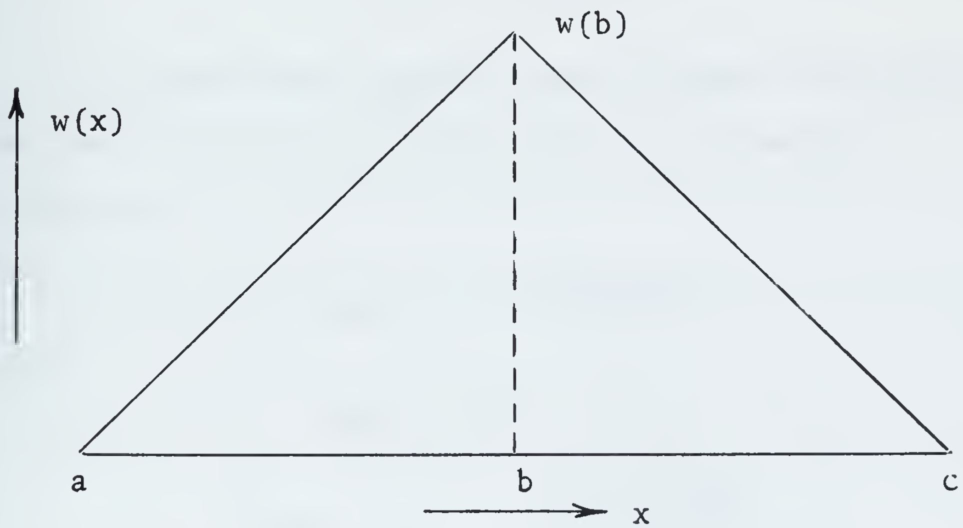


Figure 7-1B Triangular Distribution- Symmetrical

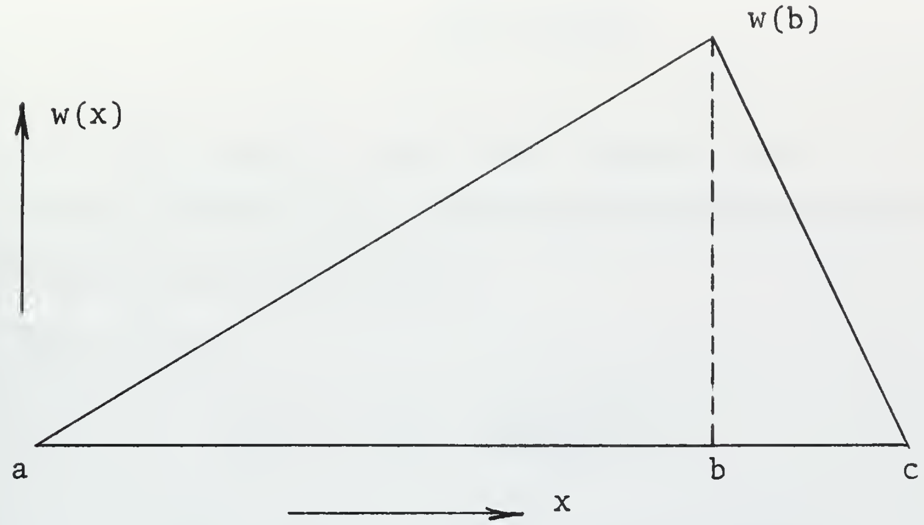


Figure 7-1C Triangular Distribution - Negative Skew

Since $\int_a^c w(x)\,dx = 1$ (= the area of the triangle) for a continuous probability distribution:

$$\int_a^c w(x)\,dx = 1/2w(b)(c-a) = 1 \tag{3}$$

from which

$$w(b) = \frac{2}{c-a} \tag{4}$$

In unpubilshed lecture notes Folkert Brons [11] proves the following properties for the triangular distribution:

$$\text{mean, } \bar{x} = \frac{a + b + c}{3} \tag{5}$$

$$\text{mode, } w(b) = \frac{2}{c - a} \tag{6}$$

$$\text{median, } x_m = c - \sqrt{\frac{(c-a)(c-b)}{2}} \tag{7}$$

for positive skew (see Figure 7-1A)

$$x_m = a + \sqrt{\frac{(c-a)(b-a)}{2}} \tag{8}$$

for negative skew (see Figure 7-1c)

Brons has also derived the cumulative distribution function [11] and has demonstrated that:

For $x \leqq b$

$$W(x < x_i) = \frac{(x_i - a)^2}{(b-a)(c-a)} \tag{9}$$

$$\text{or,} \quad x_i = a + \sqrt{W(x < x_i)(b-a)(c-a)} \tag{10}$$

For $x \leqq b$

$$W(x > x_i) = \frac{(c - x_i)^2}{(c-b)(c-a)} \tag{11}$$

$$\text{or,} \quad x_i = c - \sqrt{W(x > x_i)(c-b)(c-a)} \tag{12}$$

Further,

$$W(x \leqq b) = \frac{b-a}{c-a} \tag{13}$$

The variance for the triangular distribution is given by:

$$\sigma^2 = \frac{a^2 + b^2 + c^2 - ab - ac - bc}{18} \tag{14}$$

From which the Coefficient of Variation may be derived as follows:

$$CV = \frac{\sigma_x}{\bar{x}} = \frac{\sqrt{\dfrac{a^2 + b^2 + c^2 - ab - ac - bc}{18}}}{\dfrac{a + b + c}{3}}$$

$$CV = \frac{\sqrt{\dfrac{a^2 + b^2 + c^2 - ab - ac - bc}{2}}}{a + b + c}$$

$$CV = \frac{\sqrt{a^2 + b^2 + c^2 - ab - ac - bc}}{1.414(a + b + c)} \quad (15)$$

7.5 Simulation of the Triangular Distribution

Using the data for the highest, lowest and most likely values of discounted profit per barrel previously developed it is possible to simulate the cumulative triangular distribution of profit per barrel W(x) as follows:

Let $a = \$.50$

$b = \$.88$

$c = \$1.15$

x = specific values of expected discounted profit per barrel

then for, $\$.50 \leqq x_i \leqq \$.88$

$$x_i = a + \sqrt{W(x < x_i)(b-a)(c-a)}$$

$$x_i = .50 + \sqrt{W(x < x_i)(.88-.50)(1.15 -.50)}$$

$$x_i = .50 + \sqrt{W(x < x_i)(.2470)} \quad (16)$$

for, $\$1.15 \geqq x_i \geqq \$.88$

$$x_i = c - \sqrt{W(x > x_i)(c-b)(c-a)}$$

$$x_i = 1.15 - \sqrt{W(x > x_i)(1.15-.88)(1.15-.50)}$$

$$x_i = 1.15 - \sqrt{W(x > x_i)(.1755)} \quad (17)$$

and,

$$W(x \leqq b) = \frac{b-a}{c-a}$$

$$W(x \leqq \$.88) = \frac{.88-.50}{1.15-.50}$$

$$W(x \leqq \$.88) = .585 \qquad (18)$$

Further, from Equation 15 the Coefficient of Variation may be calculated as follows:

$$CV = \frac{\sqrt{a^2 + b^2 + c^2 - ab - ac - bc}}{1.414\ (a + b + c)} \qquad (15)$$

$$CV = \frac{\sqrt{(.50)^2+(.88)^2+(1.15)^2-(.50)(1.15)-(.88)(1.15)}}{1.414\ (.50 + .88 + 1.15)}$$

$$CV = \frac{.564}{3.580}$$

$$CV = .157$$

A series of random numbers can now be assigned to represent values of $W(x < x_i)$ from .0000 ..... to .999999...... From Equation 16, if this random number was .585 or less the random number would be substituted into Equation 10, to obtain a value of x_i. If the random number was .585 or greater

$$W(x < x_i) = 1.0 - W(x > x_i) \qquad (19)$$

would be used to obtain $W(x > x_i)$ and then that value would be substituted into Equation 12, to obtain the corresponding value of x_i. This process is illustrated in Table 7-2.

TABLE 7-2

ILLUSTRATION: PROFIT PER BARREL ASSUMING TRIANGULAR DISTRIBUTION

Random No.	$W(x < x_i)$	$W(x > x_i)$	$x_i = .50 + \sqrt{W(x < x_i)(.2470)}$	$x_i = 1.15 - \sqrt{W(x > x_i)(.1755)}$
.00	.00	(1.00)	.50	
.01	.01	(.99)	.55	
.02	.02	(.98)	.57	
.	.	.	.	
.	.	.	.	
.58	.58	(.42)	.87	
.59	(.59)	.41		.89
.60	(.60)	.40		.89
.	.	.		.
.	.	.		.
.99	(.99)	.01		1.11

In this way the best estimate of discounted profit per barrel, corresponding to a random number generated to represent a value of $W(x < x_i)$, may be simulated. For use in the Monte Carlo Simulation, a Triangular Distribution Variable Generator was developed and is included in Appendix B.

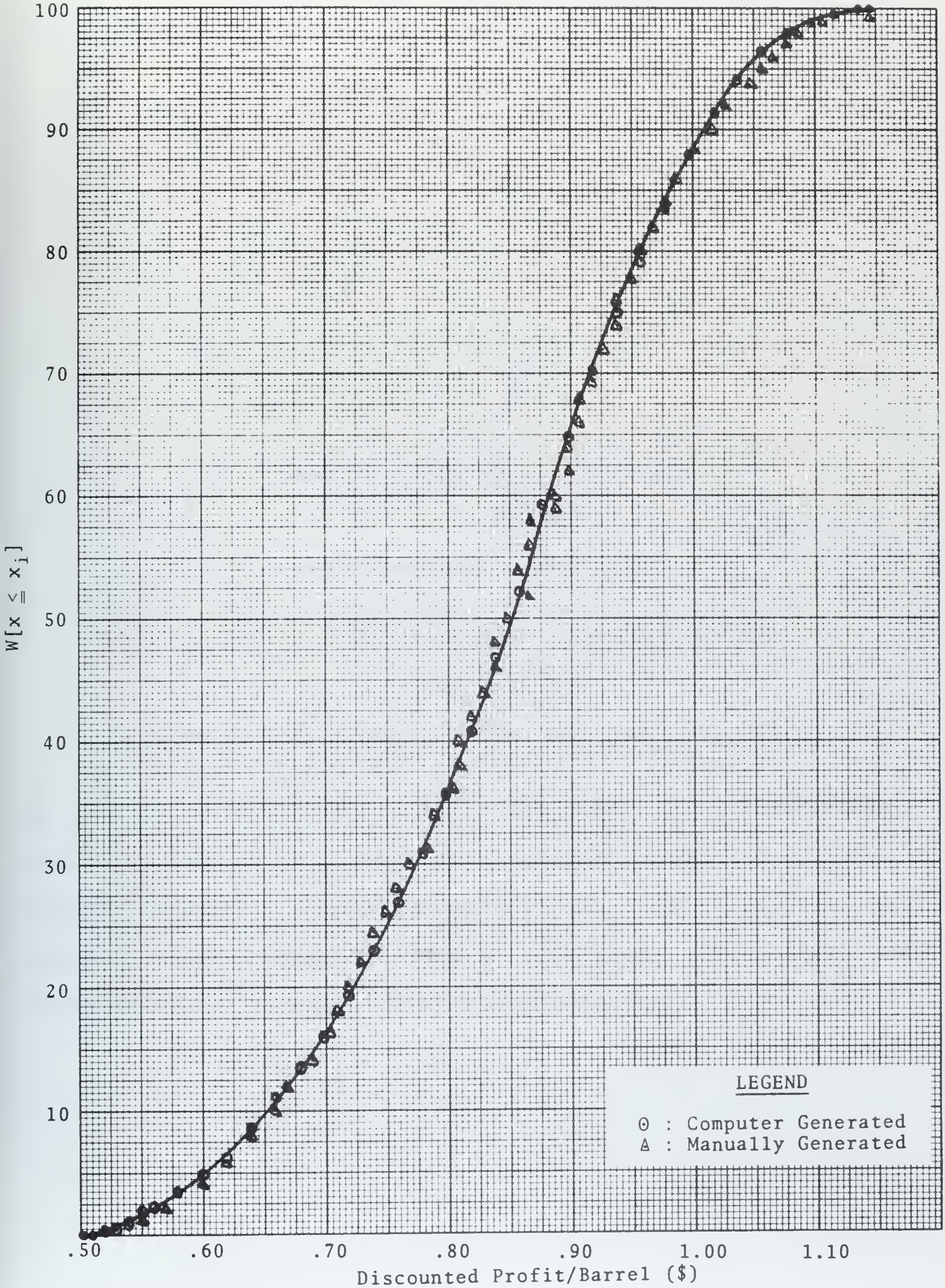
In order to test the reliability of the deviates provided by the Triangular Distribution Generator, 1000 values of the profit per barrel, e, were calculated using the program included in Appendix B and the values of a, b, and c previously

discussed. The results of this test are listed on Table 7-3 under the heading of Triangular Distribution. The values of profit per barrel thus simulated were plotted on Figure 7-2 as circles. Also included on Figure 7-2, as triangular points, are values of profit per barrel calculated manually using the technique outlined on Table 7-2. It is apparent from Figure 7-2 that there is excellent correlation between the manually calculated values and those calculated using the Triangular Distribution Generator, thus confirming the reliability of the Generator as a simulation tool. Dispite this excellent correlation one item should be recognized. By the very nature of the random number generator which generates values of numbers used to simulate $W(x < x_i)$ a value of 100% for $W(x < x_i)$ can never be achieved. Instead the highest value of $W(x < x_i)$ we can achieve using the generator is .9999..... Thus, there is no possibility that the highest value of profit per barrel (e) can ever be reached. This shortcoming is considered minor however, considering the excellent correlation for virtually the entire range of deviates.

7.6 The Beta Distribution Function

In this derivation of the Beta distribution function Brons [13] notes that a disadvantage of the Triangular distribution is that its derivate $\frac{dw(x)}{dx}$ becomes undetermined at x = a, b and c, i.e. the only points where some information about x is available. This means that at the outer limits a and b,

Figure 7-2
Cumulative Distribution Curve - Triangular Distribution



w(x) suddenly drops to zero. This is depicted in Figure 7-3.

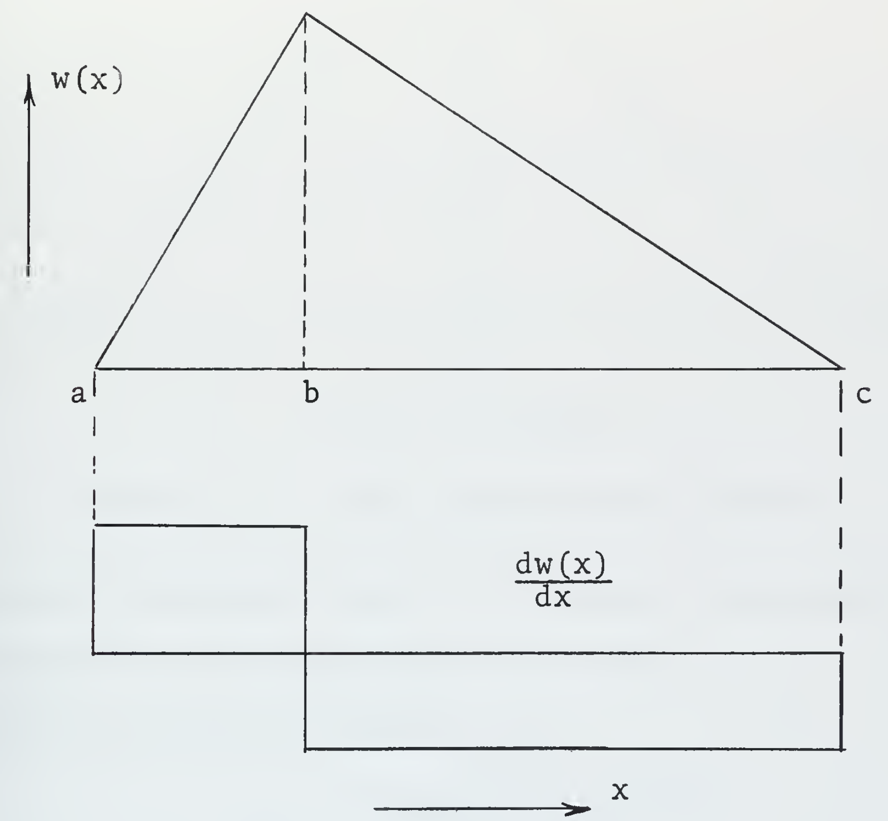


Figure 7-3 - Triangular Distribution Derivative

He also notes that a more gentle dropping off of the probability of occurence of x near the end points often better expresses reality - in that occurances in a real situation would normally taper off rather than end abruptly. This concept can be expressed by the Beta Distribution shown in Figure 7-4.

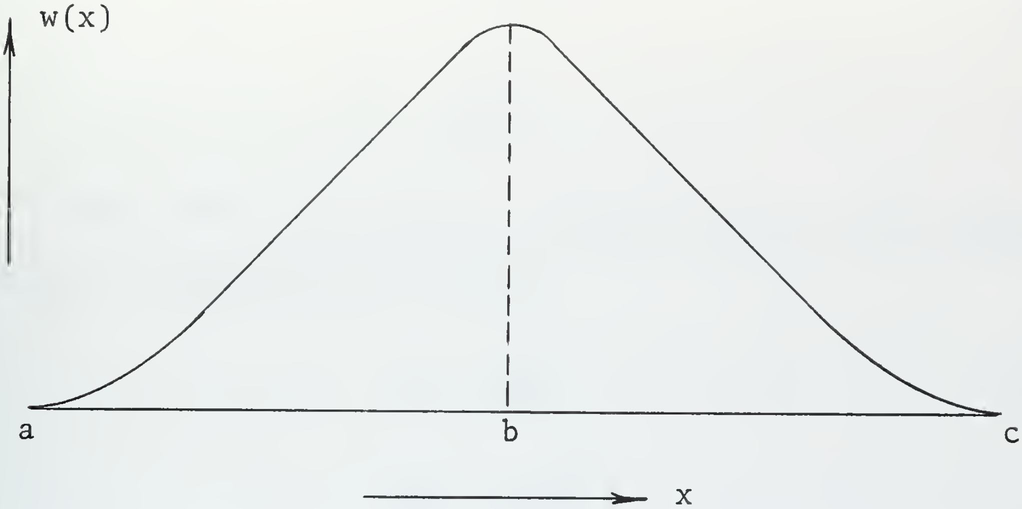


Figure 7-4 Beta Distribution Function

The Beta distribution, like the Triangular distribution, is defined by the three parameters a, b and c.
The Beta Distribution is given by

$$w(x) = \frac{\Gamma(\alpha+\beta) x^{\alpha-1} (1-x)^{\beta-1}}{\Gamma(\alpha)\ \Gamma(\beta)} \tag{20}$$

It can be seen that the Beta distribution is a ratio of gamma functions where α and β are defined by the values of a, b, and c. A derivation of the relationships between α and β and a, b and c (and of the Beta distribution itself) is beyond the scope of this thesis and can be found in several excellent discussions of this distribution [13][19].

In an unpublished report [30], W.G. Lesso developed a computer program which generates deviates having a Beta distribution. The program uses as inputs, the mean and variance of x (as expressed in a PERT type analysis):

$$\bar{x} = \frac{a + 4b + c}{6} \tag{21}$$

$$\sigma_x^2 = \frac{(c-a)^2}{36} \tag{22}$$

From which the mean and variance for the Beta distribution may be obtained as follows:

$$\bar{x}_b = \frac{\bar{x}-a}{c-a} \tag{23}$$

$$\sigma_b^2 = \frac{\sigma_x^2}{(c-a)^2} \tag{24}$$

The Lesso developed program, modified for use in this study, is included in Appendix C.

For comparision purposes the Beta Distribution Variable Generator was used to calculate 1000 values of the profit per barrel, e, using the same parameters of a,b and c as were used to calculate the profit per barrel using the Triangular Distribution Variable Generator. The values of profit per barrel thus calculated are listed in Table 7-3 under the heading of Beta Distribution. Cumulative frequency curves for the Beta and Triangular distributions, using the data contained in Table 7-3, is plotted on Figure 7-5. It can be seen from Figure 7-5 that very little difference exists between the distributions except for values at the lower end of the curves. It should be noted, however, that the mean and standard deviation for the Beta distribution was computed using the values as expressed in a PERT type analysis (see Equations 21 and 22) rather than in the form used in the Triangular distribution (see Equations 5

and 14).

Because of its relative simplicity a Triangular distribution will be assumed for the variable, profit per barrel. In the sensitivity studies which follow however, the Beta distribution will be substituted for the Triangular distribution and its effect on the venture profitability will be analyzed.

TABLE 7-3

COMPUTER SIMULATED DEVIATES FOR DISCOUNTED PROFIT PER BARREL

ASSUMING TRIANGULAR AND BETA DISTRIBUTIONS

parameters
lowest profit per barrel: $.50
most likely profit per barrel: $.88
highest profit per barrel: $1.15

	TRIANGULAR DISTRIBUTION		BETA DISTRIBUTION	
rofit/bbl ($)	Relative Frequency $w = \frac{f}{1000}$ (%)	Cumulative Frequency $W(x \leqq x_i)$ (%)	Relative Frequency $w = \frac{f}{1000}$ (%)	Cumulative Frequency $W(x \leqq x_i)$ (%)
50	0	0	0	0
51	0	0.0	0	0.0
52	0.2	0.2	0	0.0
53	0	0.2	0	0.0
54	0.5	0.7	0	0.0
55	0.9	1.6	0	0.0
56	0.6	2.2	0.1	0.1
57	0.6	2.8	0	0.1
58	0.7	3.5	0	0.1
59	0.7	4.2	0.1	0.2
60	0.7	4.9	0.3	0.5
61	0.7	5.6	0	0.5
62	0.6	6.2	0.6	1.1
63	1.1	7.3	0.6	1.7
64	1.3	8.6	0.6	2.3
65	1.4	10.0	0.8	3.1
66	1.0	11.0	1.0	4.1
67	1.3	12.3	0.9	5.0
68	1.2	13.5	1.0	6.0
69	0.8	14.3	1.4	7.4
70	1.9	16.2	1.2	8.6
71	1.9	18.1	2.0	10.6
72	1.4	19.5	2.2	12.8
73	1.7	21.2	2.0	14.8
74	1.9	23.1	1.6	16.4
75	1.9	25.0	1.8	18.2
76	2.0	27.0	2.6	20.8

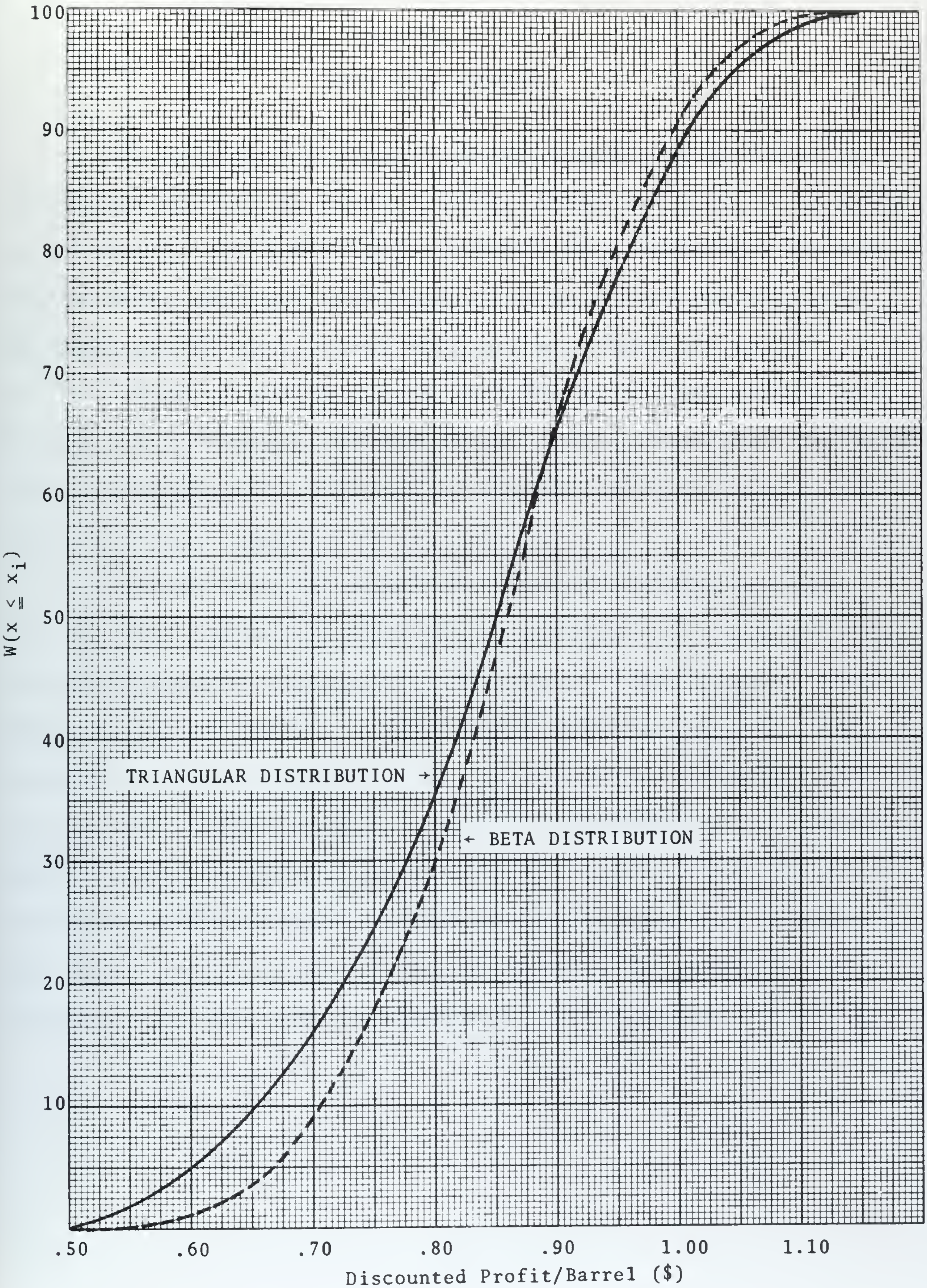
TABLE 7-3 Continued

Profit/bbl ($)	TRIANGULAR DISTRIBUTION Relative Frequency $w = \frac{f}{1000}$ (%)	TRIANGULAR DISTRIBUTION Cumulative Frequency $W(x \leqq x_i)$ (%)	BETA DISTRIBUTION Relative Frequency $w = \frac{f}{1000}$ (%)	BETA DISTRIBUTION Cumulative Frequency $W(x \leqq x_i)$ (%)
77	1.8	28.8	2.9	23.7
78	2.5	31.3	1.6	25.3
79	2.5	33.8	3.4	28.7
80	2.1	35.9	2.3	31.0
81	2.3	38.3	2.8	33.8
82	2.5	40.7	3.1	36.9
83	3.0	43.7	2.3	39.2
84	3.2	46.9	3.2	42.4
85	2.6	49.3	3.1	45.5
86	2.8	52.3	3.4	48.9
87	3.2	55.5	3.7	52.6
88	3.9	59.4	3.6	56.2
89	2.6	62.0	3.3	59.5
90	2.9	64.9	4.3	63.8
91	2.6	67.5	3.7	67.5
92	1.9	69.4	3.2	70.7
93	3.2	72.6	2.1	72.8
94	2.5	75.1	3.5	76.3
95	2.5	77.6	2.1	78.4
96	1.8	79.4	3.3	81.7
97	2.5	81.9	2.3	84.0
98	1.7	83.6	3.1	87.1
99	2.2	85.8	1.6	88.7
100	2.2	88.0	2.1	90.8
101	1.5	89.5	1.4	92.2
102	2.0	91.5	1.5	93.7
103	1.3	92.8	1.3	95.0
104	1.4	94.2	1.2	96.2
105	0.8	95.0	0.6	96.8
106	1.5	96.5	0.7	97.5
107	0.6	97.1	1.0	98.5
108	0.9	98.0	0.5	99.0
109	0.4	98.4	0.3	99.3
110	0.5	98.9	0.2	99.5

TABLE 7-3 Continued

Profit/bbl ($)	TRIANGULAR DISTRIBUTION		BETA DISTRIBUTION	
	Relative Frequency $w = \frac{f}{1000}$ (%)	Cumulative Frequency $W(x \leq x_i)$ (%)	Relative Frequency $w = \frac{f}{1000}$ (%)	Cumulative Frequency $W(x \leq x_i)$ (%)
111	0.2	99.1	0.3	99.8
112	0.5	99.6	0	99.8
113	0.4	100.0	0.2	100.0
114	0	100.0	0	100.0
115	0	100.0	0	100.0

Figure 7-5
Triangular and Beta Distribution Curves



CHAPTER VIII

DRY HOLE COSTS

In Chapter V we discussed the concept of probability of success and how it related to the number of successful wells to be expected in a drilling program. To account for the number of wells that are unsuccessful we have included dry hole, or unsuccessful well, costs into our profitability model.

While it can be argued that dry hole costs should have a continuous frequency distribution, just as the other parameters in the model, we will assume for simplicity sake that the dry hole costs are constant in this thesis.

To be consistent with the values developed in Table 7-1, we will assume that the drilling venture is to take place in the Southern Louisiana on-shore area. From January to December 1970 there were a total of 1245 wells drilled in this area with a total footage of 12,703,309 [51]. Therefore, the average well depth would be equal to 10,200 feet/well. Using 1969 average dry hole costs of $13.23/ft [50], the average dry hole cost per well would be $135,000. Allowing for inflation and deeper wells we will assume a constant dry hole cost of $150,000 for each unsuccessful well in the program.

CHAPTER IX

COMPUTER PROGRAM

The FORTRAN IV computer program, as contained herein, was developed to apply the Monte Carlo simulation technique to an exploratory well drilling venture using the concepts and relationships established in previous chapters, to determine the best estimate of discounted profit and to accumulate data for each iteration so that a cumulative probability curve can be plotted.

The program is designed to call up the same series of random numbers with each separate computer run so that sensitivity studies can be made. The random number generator used in this program is a function designed for the CDC 6600 computer system and is called by RANF(0). The generator produces pseudo random numbers, distributed uniformly in the interval $0<x<1$.

The program accepts as data: the number of wells drilled, probability of success, dry hole costs, and the value M, which is the number of end-points entered in Line 5 plus one, (Line 4); the median value of Q, the value of Q at the median + 1 standard deviation, Q84, and the boundary conditions, Q max. and Qmin., (Line 6); and the lowest, most likely and highest profit per barrel, A, B and C, (Line 7). In addition, the end points of the random number ranges corresponding to the probability of the number of successful wells is entered as NEP, (Line 5).

With this data entered, values are initialized (Lines 12, 13, 14, 15, 18, 20) and the Standard Deviation and Mean of the Lognormal distribution are calculated (Lines 16 and 17).

For each iteration specified (Line 23) the first series of operations establishes the number of successful wells (Lines 24-32). To do this a random number is called, RANF(O), multiplied by 10,000 and compared with the end points corresponding to the number of successes previously entered (Line 5). Thus, the random number called establishes the number of successes, NSW.

If the number of successes is greater than 0, the program is designed to calculate the size reserve for each success and to total them to obtain the value QSUM, which is the total reserve size discovered (Lines 37-49). For a detailed description of the Lognormal distribution generator see Appendix A.

With the total reserve size determined the profit per barrel, for this iteration, E, assuming a triangular distribution, is calculated (Lines 51-57). For a detailed discription of the Triangular distribution ganerator see Appendix B.

Having calculated or inputed all the parameters the iteration profit, PROFIT, is computed (Line 58) using the relationship expressed in Chapter III, Equation 4. Lines 59 and 60 accumulate this iteration profit and average the value, PTOT, over the number of iterations run, MM. XSQ and XSQTOT in Lines 61 and 62 are used to calculate the standard deviation and coefficient of variation later in the program.

Lines 64 through 176 are inserted to store the values of iteration profit in arrays by PROFIT intervals in order to compute the probability of the interval, PROB, and the cumulative probability, CUMP (Lines 189-202).

The variance, VAR, standard deviation, SD, and coefficient of variation, CV, are calculated for the particular set of model parameters in Lines 178, 179 and 180 using information previously determined in Lines 61 and 62.

A listing of the FORTRAN IV computer program is contained on the following pages.

```
      PROGRAM PROFIT (INPUT,OUTPUT)
      DIMENSION NEP(25), IY(40), QLOGN(20), D(40), YY(40), PROB(40),
     1 CUMP(40)
      READ 100, N,P,CU,M
      READ 110, (NEP(K), K=1,M)
      READ 111, QMEDIAN, Q84, QMAX, QMIN
      READ 112, A, B, C
      PRINT 155
      PRINT 113
      PRINT 114, N, P, CU, QMEDIAN, Q84, QMAX, QMIN
      PRINT 160, A, B, C
      PROF = 0.0
   20 PROFIT = 0.0
      XSQ = 0.0
      XSQTOT = 0.0
      SD = ALOG(Q84) - ALOG(QMEDIAN)
      TMU = ALOG(QMEDIAN)
      MM = 1
      PRINT 115
      DO 300 I = 1,40
      IY(I) = 0.0
  300 CONTINUE
      DO 99 II = 1.1000
      CALL RANF(0)
      W1 = RANF(0)
      W = W1*10000.
      J = W
      DO 10 I = 1,M
      IF (J.LE.NEP(I)) GO TO 11
   10 CONTINUE
   11 NSW = I-1
      IF (NSW.FQ.0.0) 32,40
   32 QSUM = 0.0
      E = 0.0
      GO TO 56
   40 CONTINUE
      QSUM = 0.0
      DO 45 I = 1,NSW
      RA = RANF(0)
      RB = RANF(0)
      V = (-2.0*ALOG(RA))**.5*COS(2.0*3.1416*RB)
      QNORM = V*SD + TMU
      QLOGN(I) = EXP(QNORM)
      IF (QLOGN(I)-QMIN) 46,47,48
   46 QLOGN(I) = QMIN
      GO TO 47
   48 IF (QLOGN(I)-QMAX) 47,47,49
   49 QLOGN(I) = QMAX
   47 QSUM = QSUM + QLOGN(I)
   45 CONTINUE
      CALL RANF(0)
      WXLTXI = RANF(0)
      WXGTXI = 1.0 - WXLTXI
      XI = A + SQRT(WXLTXI*(B-A)*(C-A))
      IF (XI.LE.B) 55,50
   50 XI = C - SQRT(WXGTXI*(C-B)*(C-A))
   55 E = XI/100.
   56 PROFIT = QSUM*E - (N-NSW)*CU
```

```
      PROF = PROFIT + PROF
      PTOT = PROF/MM
      XSQ = (PROFIT)**2
      XSQTOT = XSQTOT + XSQ
      PRINT II6, MM, NSW, QSUM, E, PROFIT, PTOT
      MM = MM + I
      IF(PROFIT.LT.-10.0) GO TO 61
      IF(PROFIT.LT. 0.00) GO TO 62
      IF(PROFIT.LT. 10.0) GO TO 63
      IF(PROFIT.LT. 20.0) GO TO 64
      IF(PROFIT.LT. 30.0) GO TO 65
      IF(PROFIT.LT. 40.0) GO TO 66
      IF(PROFIT.LT. 50.0) GO TO 67
      IF(PROFIT.LT. 60.0) GO TO 68
      IF(PROFIT.LT. 70.0) GO TO 69
      IF(PROFIT.LT. 80.0) GO TO 70
      IF(PROFIT.LT. 90.0) GO TO 7I
      IF(PROFIT.LT.I00.0) GO TO 72
      IF(PROFIT.LT.II0.0) GO TO 73
      IF(PROFIT.LT.I20.0) GO TO 74
      IF(PROFIT.LT.I30.0) GO TO 75
      IF(PROFIT.LT.I40.0) GO TO 76
      IF(PROFIT.LT.I50.0) GO TO 77
      IF(PROFIT.LT.I60.0) GO TO 78
      IF(PROFIT.LT.I70.0) GO TO 79
      IF(PROFIT.LT.I80.0) GO TO 80
      IF(PROFIT.LT.I90.0) GO TO 8I
      IF(PROFIT.LT.200.0) GO TO 82
      IF(PROFIT.LT.210.0) GO TO 83
      IF(PROFIT.LT.220.0) GO TO 84
      IF(PROFIT.LT.230.0) GO TO 85
      IF(PROFIT.LT.240.0) GO TO 86
      IF(PROFIT.LT.250.0) GO TO 87
      IF(PROFIT.LT.260.0) GO TO 88
      IF(PROFIT.LT.270.0) GO TO 89
      IF(PROFIT.LT.280.0) GO TO 90
      IF(PROFIT.LT.290.0) GO TO 91
      IF(PROFIT.LT.300.0) GO TO 92
      IF(PROFIT.LT.350.0) GO TO 93
      IF(PROFIT.LT.400.0) GO TO 94
      IF(PROFIT.LT.450.0) GO TO 95
      IF(PROFIT.LT.500.0) GO TO 96
      IF(PROFIT.LT.550.0) GO TO 97
      GO TO 99
61    IY(I) = IY(1) + I
      GO TO 99
62    IY(2) = IY(2) + I
      GO TO 99
63    IY(3) = IY(3) + I
      GO TO 99
64    IY(4) = IY(4) + I
      GO TO 99
65    IY(5) = IY(5) + I
      GO TO 99
66    IY(6) = IY(6) + I
      GO TO 99
67    IY(7) = IY(7) + 1
      GO TO 99
```

```
68   IY(8) = IY(8) + 1
     GO TO 99
69   IY(9) = IY(9) + 1
     GO TO 99
70   IY(10) = IY(10) + 1
     GO TO 99
71   IY(11) = IY(11) + 1
     GO TO 99
72   IY(12) = IY(12) + 1
     GO TO 99
73   IY(13) = IY(13) + 1
     GO TO 99
74   IY(14) = IY(14) + 1
     GO TO 99
75   IY(15) = IY(15) + 1
     GO TO 99
76   IY(16) = IY(16) + 1
     GO TO 99
77   IY(17) = IY(17) + 1
     GO TO 99
78   IY(18) = IY(18) + 1
     GO TO 99
79   IY(19) = IY(19) + 1
     GO TO 99
80   IY(20) = IY(20) + 1
     GO TO 99
81   IY(21) = IY(21) + 1
     GO TO 99
82   IY(22) = IY(22) + 1
     GO TO 99
83   IY(23) = IY(23) + 1
     GO TO 99
84   IY(24) = IY(24) + 1
     GO TO 99
85   IY(25) = IY(25) + 1
     GO TO 99
86   IY(26) = IY(26) + 1
     GO TO 99
87   IY(27) = IY(27) + 1
     GO TO 99
88   IY(28) = IY(28) + 1
     GO TO 99
89   IY(29) = IY(29) + 1
     GO TO 99
90   IY(30) = IY(30) + 1
     GO TO 99
91   IY(31) = IY(31) + 1
     GO TO 99
92   IY(32) = IY(32) + 1
     GO TO 99
93   IY(33) = IY(33) + 1
     GO TO 99
94   IY(34) = IY(34) + 1
     GO TO 99
95   IY(35) = IY(35) + 1
     GO TO 99
96   IY(36) = IY(36) + 1
     GO TO 99
```

```
97   IY(37) = IY(37) + 1
99   CONTINUE
60   MM = MM - 1
     VAR = (XSQTOT - (MM*(PTOT**2)))/(MM - 1)
     SD = SQRT(VAR)
     CV = SD/PTOT
     PRINT 140, MM,PTOT
     PRINT 141, VAR, SD, CV
     PRINT 144
     PRINT 155
     PRINT 113
     PRINT 114, N, P, CU, QMEDIAN, Q84, QMAX, QMIN
     PRINT 160, A, B, C
     PRINT 145
     SUMM = 0.0
     D(1) = -10.0
     DO 200 IM = 1,37
     YY(IM) = IY(IM)
     PROB(IM) = YY(IM)/1000.
     SUMM = SUMM + PROB(IM)
     CUMP(IM) = SUMM
     KJ = IM + 1
     IF(IM-32) 210, 220, 220
210  D(KJ) = D(IM) + 10.0
     GO TO 230
220  D(KJ) = D(IM) + 50.0
230  CONTINUE
     PRINT 150, D(IM), PROB(IM), CUMP(IM)
200  CONTINUE
100  FORMAT (10X,I5,2F10.3,I5)
110  FORMAT (10X,17I4)
111  FORMAT (10X,4F10.3)
112  FORMAT (10X,3F10.2)
113  FORMAT (///,6X,*NO. OF WELLS*,3X,*PROB OF*,2X,*DRY HOLE*,/,9X,*DRI
    1LLED*,5X,*SUCCESS*,4X,*COST*,4X,*QMEDIAN*,3X,*Q84*,5X,*QMAX*,5X,
    2*QMIN*)
114  FORMAT (10X,I5,8X,F4.3,4X,F4.3 ,3X,F7.2,2X,F7.2,2X,F7.2,2X,F7.2//)
115  FORMAT(5X,*ITERATION*,3X,*NUMBER OF*,4X,*FIELD*,2X,*PROFIT PER*,2X
    1,*ITERATION*,3X,*BEST ESTIMATE*,/17X,*SUCCESSES*,4X,*SIZES*,5X,*BA
    2RREL*,6X,*PROFIT*,6X,*OF PROFIT*,//)
116  FORMAT (  5X,I4,11X,I3  ,6X,F6.2,6X,F4.2,7X,F6.2,8X,F6.2)
140  FORMAT (///,5X,*THE BEST ESTIMATE OF PROFIT BY THE MONTE CARLO SIM
    1ULATION METHOD*,/7X,*AT *,I4(* ITERATIONS IS *,F7.2,* MILLION DOLL
    1ARS*)
141  FORMAT (///,5X,*THE VARIANCE IS *,F12.3,/,5X,*THE STANDARD DEVIATI
    1ON IS *,F12.3,/,5X,*THE COEFFICIENT OF VARIATION IS *,F6.4)
144  FORMAT (1H1)
145  FORMAT    (15X,*UPPER LIMIT OF*,7X,*PROBABILITY*,5X,*CUMMULATIVE*,
    1/,15X,*PROFIT INTERVAL*,5X,*OF INTERVAL*,5X,*PROBABILITY*,///)
150  FORMAT (20X,F4.0,15X,F5.4,11X,F5.4)
155  FORMAT (1H1,15X,*PROFIT DISTRIBUTION OF AN EXPLORATORY DRILLING PR
    1OGRAM*)
160  FORMAT (/15X,*THE TRIANGULAR DISTRIBUTION FOR DETERMINATION OF THE
    1*,/15X,*PROFIT PER BARREL HAS THE FOLLOWING EXTREMES IN CENTS - *,
    2/,15X,*A = *,F6.2,7X,*B = *,F6.2,7X,*C = *,F6.2,///)
     END
```

CHAPTER X

BEST ESTIMATE OF DISCOUNTED EXPECTED PROFIT USING BASE DATA

In previous chapters the profitability model was developed and the individual parameters and their frequency distributions were discussed. In addition, certain Base Data was established for each parameter so that the best estimate of discounted expected profit, E(P), could be calculated. This Base Data is summarized in Table 10-1.

TABLE 10-1

BASE DATA

Parameter	Distribution	Value
Number of Wells Drilled - n	-	20
Probability of Success - p	Binomial	.15
Reservoir Size	Lognormal	
Q at 5% probability		5 x 10^6 bbl
Q at 95% probability		79 x 10^6 bbl
Q maximum		140 x 10^6 bbl
Q minimum		1 x 10^6 bbl
Discounted Profit/Barrel	Triangular	
Lowest - e_a		$ .50/bbl
Most Likely - e_b		$ .88/bbl
Highest - e_c		$1.15/bbl
Dry Hole Cost - C_u	None	$ 150,000
Number of Iterations		1,000

On the basis of this Base Data and using the Fortran IV program included in Chapter IX a value of Best Estimate of Discounted Expected Profit, E(P), of $68.02 million was obtained. See Appendix D for a printout of the number of successes, reserve sizes, profit per barrel, iteration profit and average best estimate of discounted profit for each of the 1000 iterations using the Base Data. In addition, Appendix D contains cumulative probability data for each specified interval of E(P). Using this cumulative probability information the histogram included as Figure 10-1 was prepared from which it can be seen that:

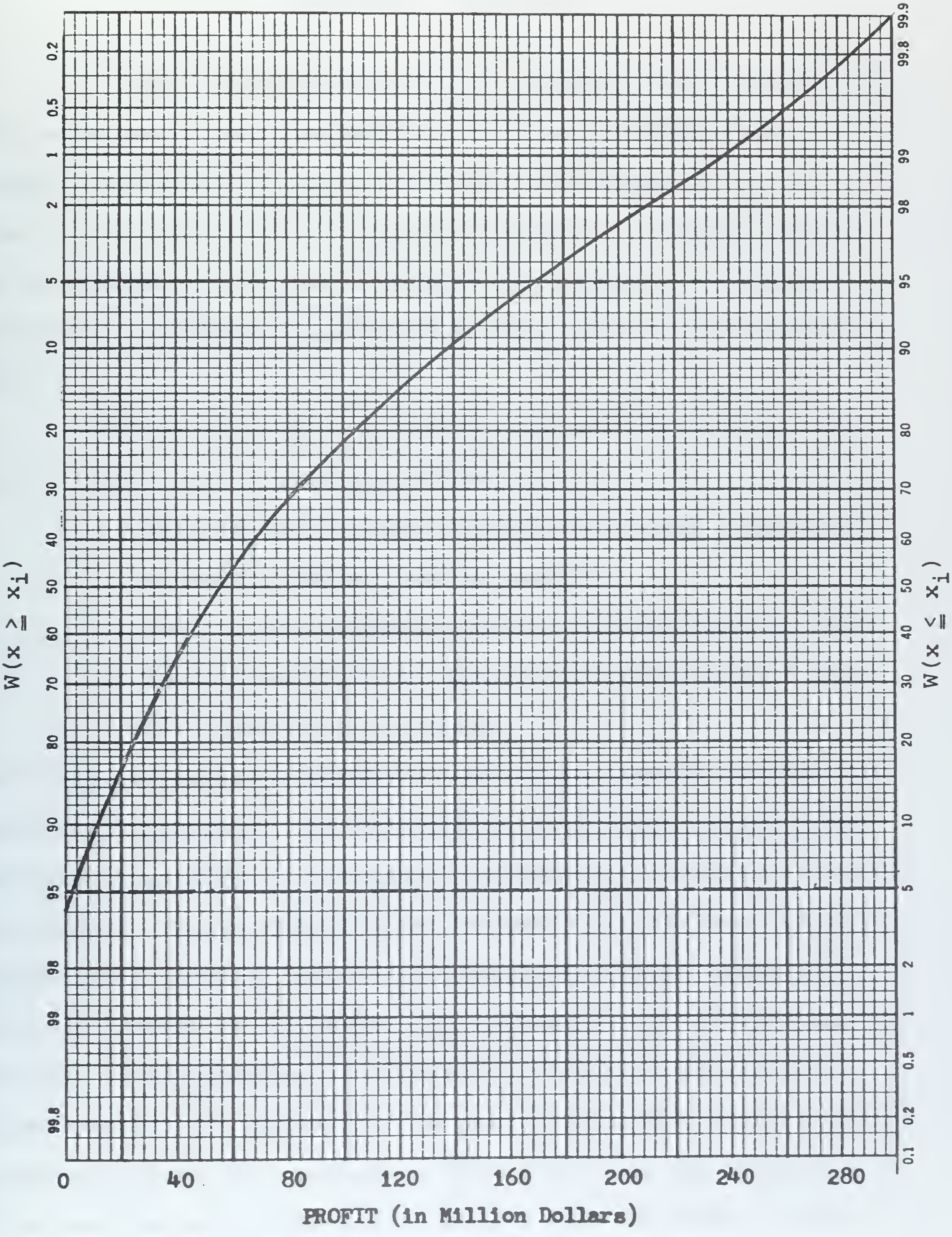
1. The Median Value (at 50% probability) of Discounted Profit is $56.03 million.
2. With 90% confidence we could expect a discounted profit of between $1.51 million and $170.0 million.
3. The probability of a loss is 4.2%, or stating it another way, there is a 95.8% chance of a profit.

Further, from the printout in Appendix D we see that:

4. The standard deviation is $53.66 million.
5. The coefficient of variation is .7889.

The significance of the standard deviation and coefficient of variation as measures of risk were discussed in Chapter IV.

Figure 10-1
Cumulative Probability - Base Data



CHAPTER XI

SENSITIVITY ANALYSIS

Freedom from uncertainty is a luxury rarely enjoyed by the contemporary decision maker. In a society characterized by change, uncertainty is an established and accepted fact of life. This is especially true in the oil business. Notwithstanding the difficulties, the decision maker is expected to attain satisfactory results as measured by the goals of his organization.

In the face of uncertainty, the most recurring questions he faces are of the form, "What if...?

1. What if labor costs rise and dry holes cost twice as much? Three times as much?
2. What if the posted price of oil decreases by 30%?
3. What if we drill 30 exploratory wells rather than 20? How about 10 wells?

The "what if" question may be viewed as an introduction to ths sensitivity analysis. In its mathematical sense, sensitivity analysis is a study to determine how possible changes or errors in parameter values affect model outputs. In its applied organizational setting, sensitivity analysis may be broadly defined as a study to determine the responsiveness of the conclusions of an analysis to changes or errors in parameter values used in the analysis. In other words, would our decision be any different if a parameter value, such as the probability of success in out profitability model, were to change. The

prime function of sensitivity analysis can be said to be to provide a better understanding of risk. Sensitivity analysis can provide guidelines for allocating organizational resources to data collection and data refinement activities [40]. The benefits of subjecting models to sensitivity analysis are summarized by Arnoff and Netzorg [2]:

> The use of an operations research model is especially important and advantageous in that it permits experimentation "on paper", without manipulation of the actual system. In using the model, one can assess the sensitivity (response) of the system to a wide variety of conditions - without requiring either the time, expense, or risks associated with experimenting with the system itself (if such experimentation would, in fact, be possible and meaningful). Hidden relationships can be brought to light and brought to bear upon decisions and control of activity.

In this thesis a profitability model for an exploratory well drilling venture was developed and tested with Base parameters. A decision problem is now faced. If the model was developed in order to provide results which would serve as a basis for recommending action to management, it is possible, at this point, to use the model results as is, or to select other alternatives: to collect additional data; refine initial parameters; or even alter the model itself. On the significance of this decision, Willian T. Morris [35] comments:

> The nature of management science is such that sooner or later it has to recommend some action other than making more observations. Usually this comes sooner rather than later. Eventually it will suggest that action be taken consistent with either acceptence or rejection of the hypothesis. It will recommend management "act as if" the hypothesis were true or "as if" it were false. Indeed, the fundamental question in management science may well be when to stop collecting data and

> developing the model, and when to produce a recommendation for action.

In the past the emphasis has been on whether information required for decision making could be developed. With our growing capacity to generate information or data, the emphasis now appears to be shifting to the question as to what extent this capacity can be utilized beneficially. It may be recognized that while the collection and refinement of data are costly, it may serve to improve recommentations for action. In most cases, however, little basis exists for projecting whether the additional information is worth the cost. Statistical decision theory, and a model such as the one developed in this thesis, will assist in the decision as to whether additional information is "worth the cost". The basic requirements for the statistical decision theory model are summarized by Morris [36] as follows:

> [The Model] must be able to enumerate the possible outcomes of future data collection efforts and, further, to compute the probabilities of these outcomes. In addition, it must be possible to indicate just how the information willquantitatively change the decision maker's view of his choice.

In an excellent example, which illustrates this concept, C. Jackson Grayson [21], demonstrates the application of decision theory to drilling decisions by oil and gas operators and also to the decision problem of whether to purchase additional information before making a final drilling decision.

Clearly then, a sensitiviey analysis which discloses decision insensitivity to estimated parameter variations

precludes the necessity of introducing more costly, time consuming data, collection data refinement and statistical decision models into the analysis.

It is in this vein that the sensitiviey studies are conducted in this thesis. The profitability model provides the necessary vehicle for the analysis. From the results it is possible to determine the outcomes, as well as probabilities of these outcomes, associated with changes in parameter values thus providing some answers to the "What if?" questions posed at the beginning of this chapter.

CHAPTER XII

SENSITIVITY STUDIES

12.1 Outline of Studies

The sensitivity studies were conducted using the profitability model developed in Chapter III. In these studies each parameter in the model was varied from the base values as summarized in Table 10-1. Each time a parameter was varied all remaining parameters were set to their base values and the FORTRAN IV program, developed in Chapter IX, was executed 1000 times. In this manner a printout similar to the one for the Base data, included in Appendix D, was obtained for each parameter change or Case.

Thus, for each Case, at 1000 iterations, we have the best estimate of discounted profit, the standard deviation, the coefficient of variation, the iteration profit, cumulative probability data, etc. Tables 12-1 through 12-9, grouped by parameter being varied, contain a summary of some of this data (at 1000 iterations) as well as extrapolated information such as the median discounted profit, the maximum and minimum discounted profits within 90% confidence limits and the chance of making a profit greater than zero. The values using the Base data have been included on each Table to assist in evaluation. It should be noted that each time the model was run the same series of random numbers were called up so that comparisons between Cases could be made. Figures 12-1 through 12-8 contain histograms for several Cases, plotting the cum-

ulative probability for each specified interval of discounted expected profit. Figures 12-1 through 12-8 are similar in presentation to Figure 10-1 for the Base data. On each histogram the 90% Confidence Limits have been represented by dashed lines at 5% and 95% cumulative probabilities.

To assist in the comparison of each Case to the Base, Tables 12-1A through 12-9A have been prepared and are inserted behind the Table it serves to amplify. These Tables contain the percentage differences between the Case parameter set values and the Base parameter set values. The percentages were calculated using the following relationship:

$$\Delta(\text{in }\%) = \frac{(\text{Case data value}) - (\text{Base data value})}{(\text{Base data value})} \times 100, \quad (1)$$

In addition to the Cases noted above, the Base data was run for 10,000 iterations rather than 1000 iterations. Additionally, the Beta Distribution Variable Generator was substituted for the triangular distribution of profit/barrel in the FORTRAN IV Program and the Base data was run for 1000 iterations. The results of both these variations are summarized on Table 12-10.

12.2 General Observations

One of the prime purposes of the sensitivity studies is to assess the change in observed values resulting from a change in parameter value. The majority of the effects of

TABLE 12-1

VARIATIONS IN NUMBER OF WELLS DRILLED - n

Parameters

p = 15%
Q(5%) = 5; Q(95%) = 79; Q(max) = 140; Q(min) = 1; (in 10^6bbl)
e(a) = \$.50; e(b) = \$.88; e(c) = \$1.15;
C_u = \$.15 million

NOTE: Profit (\$x10^6)

Case No.	No. of Wells Drilled (n)	Best Estimate of Discounted Profit - E(P) (mean)	Discounted Profit (median)	Discounted Profit with 90% Confidence Limits Min.	Max.	Standard Deviation (σ)	Coeff. of Variation (cv)	Chance of a Profit > Zero (%)
Base	20	68.02	56.03	1.51	170.00	53.66	.7889	95.8
1	5	15.80	4.03	(-8.87)	73.33	25.17	1.5926	55.8
2	10	33.71	22.96	(-7.24)	110.00	37.52	1.1132	81.9
3	12	39.49	27.21	(-6.71)	125.71	42.38	1.0734	84.8
4	15	51.48	38.52	(-4.25)	146.00	47.35	.9195	91.3
5	18	60.02	46.74	(-1.38)	155.79	50.44	.8403	94.2
6	22	73.52	62.11	3.65	182.67	55.09	.7494	96.9
7	25	83.12	69.78	9.38	195.33	58.71	.7063	98.0
8	30	99.47	87.35	16.29	222.00	63.36	.6369	99.3
9	35	115.37	102.17	22.73	249.38	71.80	.6223	99.3

TABLE 12-1A

VARIATION FROM BASE DATA - PARAMETER n

n = 20

Case No.	No. of Wells Drilled (n)	Variation of Parameter From Base (%)	Change in Best Estimate of Discounted Profit - E(P) (%)	Change in Discounted Profit with 90% Confidence Limits (%) Min.	Max.	Change in Standard Deviation (σ) (%)	Change in Coeff. of Variation (cv) (%)	Change in Chance of a Profit > Zero (%)
1	5	-75.00	-76.77	-687.42	-56.86	-53.09	101.88	-41.75
2	10	-50.00	-50.44	-597.47	-35.29	-30.06	41.11	-14.51
3	12	-40.00	-41.94	-544.37	-26.05	-21.02	36.06	-11.48
4	15	-25.00	-24.32	-380.79	-14.12	-11.78	16.55	-4.70
5	18	-10.00	-11.76	-191.39	-8.36	-6.00	6.52	-1.67
6	22	10.00	8.09	141.72	7.45	2.66	-5.01	1.15
7	25	25.00	22.20	521.19	14.90	9.41	-10.47	2.30
8	30	50.00	46.24	978.81	30.59	18.08	-19.27	3.65
9	35	75.00	69.61	1405.30	46.69	33.81	-21.12	3.65

TABLE 12-2

VARIATIONS IN PROBABILITY - p

Parameters

$n = 20$
$Q(5\%) = 5$; $Q(95\%) = 79$; $Q(max) = 140$; $Q(min) = 1$; (in 10^6bbl)
$e(a) = \$.50$; $e(b) = \$.88$; $e(c) = \$1.15$;
$C_u = \$.15$ million

NOTE: Profit ($\$x10^6$)

Case No.	Prob. of Success (p)	Best Estimate of Discounted Profit - E(P) (mean)	Discounted Profit (median)	Discounted Profit with 90% Confidence Limits Min.	Max.	Standard Deviation (σ)	Coeff. of Variation (cv)	Chance of a Profit > Zero (%)
Base	15%	68.02	56.03	1.51	170.00	53.66	.7889	95.8
10	5	18.19	7.45	(-8.68)	78.57	29.13	1.6020	62.0
11	8	33.75	21.42	(-7.34)	116.92	39.37	1.1664	81.2
12	10	43.56	31.90	(-6.30)	133.64	44.21	1.0149	86.5
13	18	81.08	69.34	7.50	190.77	58.04	.7158	98.0
14	20	90.46	78.50	13.50	207.89	60.18	.6653	99.0
15	22	100.50	88.36	17.93	216.15	63.86	.6354	99.4
16	25	114.13	98.68	23.33	251.11	71.55	.6269	99.9
17	27	125.12	113.65	30.57	260.71	70.07	.5600	99.9
18	30	137.27	124.82	36.77	274.37	74.60	.5453	100.0

TABLE 12-2A

VARIATION FROM BASE DATA - PARAMETER p

p = 15%

Case No.	Prob. of Success (p) (%)	Variation of Parameter From Base (%)	Change in Best Estimate of Discounted Profit - E(P) (%)	Change in Discounted Profit with 90% Confidence Limits (%) Min.	Max.	Change in Standard Deviation (σ) (%)	Change in Coeff. of Variation (cv) (%)	Change in Chance of a Profit > Zero (%)
10	05	-66.67	-73.26	-674.83	-53.78	-45.71	103.09	-35.28
11	08	-46.67	-50.38	-586.09	-31.22	-26.63	47.85	-15.24
12	10	-33.33	-35.96	-517.22	-21.39	-17.61	28.65	-9.71
13	18	20.00	19.20	396.69	12.22	8.16	-9.27	2.30
14	20	33.33	32.99	794.04	22.29	12.15	-15.67	3.34
15	22	46.67	47.60	1087.42	27.15	19.01	-19.46	3.76
16	25	66.67	67.79	1445.03	47.71	33.34	-20.53	4.28
17	27	80.00	83.95	1924.50	53.36	30.58	-29.02	4.28
18	30	100.00	101.81	2335.10	61.39	39.02	-31.12	4.38

TABLE 12-3

VARIATIONS IN Q AT 5% PROBABILITY

Parameters

$n = 20$
$p = .15$
$Q(95\%) = 79$; $Q(max) = 140$; $Q(min) = 1$; (in 10^6bbl)
$e(a) = \$.50$; $e(b) = \$.88$; $e(c) = \$1.15$
$C_u = \$.15$ million

NOTE: Profit ($\$x10^6$)

Case No.	Q at 5% Prob. (10^6bbl)	Best Estimate of Discounted Profit - E(P) (mean)	Discounted Profit (median)	Discounted Profit with 90% Confidence Limits Min.	Max.	Standard Deviation (σ)	Coeff. of Variation (cv)	Chance of a Profit Zero >(%)
Base	5.0	68.02	56.03	1.51	170.00	53.66	.7889	95.8
19	1.0	46.15	30.34	(-3.42)	148.89	48.92	1.0600	92.4
20	3.5	61.16	48.00	0.56	164.29	51.82	.8473	95.4
21	4.0	63.24	49.89	0.98	165.00	52.09	.8236	95.6
22	4.5	65.30	53.77	1.38	167.57	52.54	.8047	95.8
23	5.5	69.34	57.65	1.60	172.86	53.80	.7758	95.8
24	6.0	70.90	59.48	1.96	174.44	53.86	.7597	95.9
25	6.5	72.89	62.34	2.09	180.00	54.66	.7500	95.9
26	9.0	80.42	70.14	3.60	189.33	56.88	.7072	95.9

TABLE 12-3A

VARIATION FROM BASE DATA - PARAMETER Q at 5%

Q at 5% = 5 x 10^6 bbl

Case No.	Q at 5% (10^6bbl)	Variation of Parameter From Base (%)	Change in Best Estimate of Discounted Profit - E(P) (%)	Change in Discounted Profit with 90% Confidence Limits (%) Min.	Max.	Change in Standard Deviation (σ) (%)	Change in Coeff. of Variation (cv) (%)	Change in Chance of a Profit > Zero (%)
19	1.0	-80.00	-32.15	-326.49	-12.42	-8.84	34.36	-3.55
20	3.5	-30.00	-10.09	-62.91	-3.36	-3.43	7.40	-.42
21	4.0	-20.00	-7.03	-35.10	-2.94	-2.92	4.40	-.21
22	4.5	-10.00	-4.00	-8.61	-1.43	-2.09	2.00	0
23	5.5	10.00	1.94	5.96	1.68	.26	-.40	0
24	6.0	20.00	4.23	29.80	2.61	.37	-2.47	.10
25	6.5	30.00	7.16	38.41	5.88	1.86	-4.93	.10
26	9.0	80.00	18.23	138.41	11.37	6.00	-10.36	.10

TABLE 12-4

VARIATIONS IN Q AT 95% PROBABILITY

Parameters

$n = 20$
$p = .15$
$Q(5\%) = 5$; $Q(max) = 140$; $Q(min) = 1$; (in 10^6bbl)
$e(a) = \$.50$; $e(b) = \$.88$; $e(c) = \$1.15$
$C_u = \$.15$ million

NOTE: Profit ($\$x10^6$)

Case No.	Q at 95% Prob. (10^6bbl)	Best Estimate Of Discounted Profit - E(P) (mean)	Discounted Profit (median)	Discounted Profit with 90% Confidence Limits Min.	Max.	Standard Deviation (σ)	Coeff. of Variation (cv)	Chance of a Profit > Zero (%)
Base	79.0	68.02	56.03	1.51	170.00	53.66	.7889	95.8
27	15.8	23.45	21.39	0.57	54.41	15.90	.6781	95.9
28	55.3	52.42	44.14	1.25	128.33	40.37	.7702	95.8
29	63.2	57.50	48.19	1.38	142.73	44.36	.7714	95.8
30	71.1	62.64	52.37	1.40	158.75	49.17	.7849	95.8
31	86.9	72.63	58.97	1.57	183.00	57.49	.7916	95.8
32	94.8	77.14	63.65	1.60	190.00	61.13	.7925	95.8
33	102.7	81.69	66.67	1.63	202.22	65.06	.7964	95.8
34	140.0	97.46	80.47	1.90	236.67	76.56	.7855	95.8

TABLE 12-4A

VARIATION FROM BASE DATA - PARAMETER Q at 95%

Q at 95% = 79 x 10^6 bbl

Case No.	Q at 95% Prob. (10^6bbl)	Variation of Parameter From Base (%)	Change in Best Estimate of Discounted Profit - E(P) (%)	Change in Discounted Profit with 90% Confidence Limits (%) Min.	Max.	Change in Standard Deviation (σ) (%)	Change in Coeff. of Variation (cv) (%)	Change in Chance of a Profit > Zero (%)
27	15.8	-80.00	-65.52	-62.25	-67.99	-70.37	-14.04	0
28	55.3	-30.00	-22.93	-17.22	-24.51	-24.77	-2.37	0
29	63.2	-20.00	-15.47	-8.61	-16.04	-17.33	-2.22	0
30	71.1	-10.00	-7.91	-7.28	-6.62	-8.37	-.51	0
31	86.9	10.00	6.78	3.97	7.65	7.14	.34	0
32	94.8	20.00	13.41	5.96	11.76	13.92	.46	0
33	102.7	30.00	20.10	7.95	18.95	21.24	.95	0
34	140.0	77.22	43.28	25.83	39.22	42.68	-.43	0

TABLE 12-5

VARIATIONS IN Q BOUNDARY LIMITS

Parameters

$n = 20$
$p = .15$
$Q(5\%) = 5$; $Q(95\%) = 79$; (in 10^6 bbl)
$e(a) = \$.50$; $e(b) = \$.88$; $e(c) = \$1.15$
$C_u = \$.15$ million

NOTE: Profit ($\$ \times 10^6$)

Case No.	Boundary Cond.-Q (10^6 bbl) Max.	Min.	Best Estimate of Discounted Profit - E(P) (mean)	Discounted Profit (median)	Discounted Profit with 90% Confidence Limits Min.	Max.	Standard Deviation (σ)	Coeff. of Variation (cv)	Chance of a Profit > Zero (%)
Base	140	1.0	68.02	56.03	1.51	170.00	53.66	.7889	95.8
35	79	1.0	64.35	55.60	1.51	153.89	47.33	.7356	95.8
36	280	1.0	69.29	56.03	1.51	178.00	57.70	.8236	95.8
37	140	0.5	68.02	56.03	1.51	170.00	53.66	.7889	95.8
38	140	2.0	68.02	56.03	1.51	170.00	53.66	.7889	95.8
39	140	5.0	68.18	56.12	1.70	170.00	53.68	.7874	95.9

TABLE 12-5A

VARIATION FROM BASE DATA - PARAMETER Q(max), Q(min)

Boundary Conditions Q(max) = 140; Q(min) = 1 (in bbl)

Case No.	Boundary Cond.-Q (10^6 bbl) Max.	Min.	Variation of Parameter From Base (%) Max.	Min.	Change in Best Estimate of Discounted Profit - E(P) (%)	Change in Discounted Profit with 90% Confidence Limits (%) Min.	Max.	Change in Standard Deviation (σ) (%)	Change in Coeff. of Variation (cv) (%)	Change in Chance of a Profit > Zero (%)
35	79	1.0	-43.57	0	-5.40	0	-9.48	-11.80	-6.76	0
36	280	1.0	100.00	0	1.87	0	4.71	7.53	5.54	0
37	140	.5	0	-50	0	0	0	0	0	0
38	140	2.0	0	100	0	0	0	0	-.01	0
39	140	5.0	0	400	.24	12.58	0	.04	-.19	.10

TABLE 12-6

VARIATIONS IN DRY HOLE COST - C_u

Parameters

n = 20
p = .15
Q(5%) = 5; Q(95%) = 79; Q(max) = 140; Q(min) = 1; (in 10^6bbl)
e(a) = $.50; e(b) = $.88; e(c) = $1.15

NOTE: Profit ($\$x10^6$)

Case No.	Dry Hole Cost ($\$x10^6$)	Best Estimate of Discounted Profit - E(P) (mean)	Discounted Profit (median)	Discounted Profit with 90% Confidence Limits Min.	Max.	Standard Deviation (σ)	Coeff. of Variation (cv)	Chance of a Profit > Zero (%)
Base	.150	68.02	56.03	1.51	170.00	53.66	.7889	95.8
40	.075	69.30	57.32	2.00	174.00	53.58	.7732	95.9
41	.100	68.87	56.79	1.76	174.00	53.60	.7783	95.9
42	.200	67.17	55.25	0.93	170.00	53.72	.7997	95.5
43	.300	65.47	53.78	(-0.74)	170.00	53.83	.8221	94.6
44	.500	62.07	49.52	(-3.24)	167.50	54.05	.8707	92.6
45	1.000	53.58	42.29	(-10.00)*	160.00	54.61	1.0193	86.8

* at 7.10%

TABLE 12-6A

VARIATIONS FROM BASE DATA - PARAMETER C_u

C_u = \$.150 million

Case No.	Dry Hole Cost (\$ x10^6)	Variation of Parameter From Base (%)	Change in Best Estimate of Discounted Profit - E(P) (%)	Change in Discounted Profit with 90% Confidence Limits (%) Min.	Max.	Change in Standard Deviation (σ) (%)	Change in Coeff. of Variation (cv) (%)	Change in Chance of a Profit > Zero (%)
40	.075	-50.00	1.88	32.45	2.35	-.15	-1.99	.10
41	.100	-33.33	1.25	16.56	2.35	-.11	-1.34	.10
42	.200	33.33	-1.25	-38.41	0	.11	1.37	-.31
43	.300	100.00	-3.75	-149.01	0	.32	4.21	-1.25
44	.500	233.33	-8.75	-314.57	-1.47	.73	10.37	-3.34
45	1.000	566.67	-21.23	-762.25	-5.88	1.77	29.21	-9.39

TABLE 12-7

VARIATIONS IN LOWEST VALUE OF PROFIT/BARREL - e(a)

Parameters

$n = 20$
$p = .15$
$Q(5\%) = 5$; $Q(95\%) = 79$; $Q(max) = 140$; $Q(min) = 1$; (in 10^6bbl)
$e(b) = \$.88$; $e(c) = \$1.15$
$C_u = \$.15$

NOTE: Profit ($\$x10^6$)

Case No.	Lowest Profit/bbl e(a) ($)	Best Estimate of Discounted Profit - E(P) (mean)	Discounted Profit (median)	Discounted Profit with 90% Confidence Limits Min.	Max.	Standard Deviation (σ)	Coeff. of Variation (cv)	Chance of a Profit > Zero (%)
Base	.50	68.02	56.03	1.51	170.00	53.66	.7889	95.8
46	.25	60.99	48.39	0.98	157.69	50.76	.8332	95.6
47	.35	63.80	51.92	1.36	161.67	51.79	.8117	95.8
48	.40	65.21	53.21	1.43	166.25	52.37	.8031	95.8
49	.45	66.61	54.87	1.43	167.78	52.99	.7955	95.8
50	.55	69.43	57.16	1.54	176.67	54.37	.7830	95.8
51	.60	70.83	58.21	1.57	180.83	55.11	.7880	95.8
52	.65	72.24	60.43	1.70	181.82	55.89	.7736	95.8

TABLE 12-7A

VARIATION FROM BASE DATA - PARAMETER e(a)

e(a) = $.50

Case No.	Lowest Profit/bbl e(a) (%)	Variation of Parameter From Base (%)	Change in Best Estimate of Discounted Profit - E(P) (%)	Change in Discounted Profit with 90% Confidence Limits (%) Min.	Max.	Change in Standard Deviation (σ) (%)	Change in Coeff. of Variation (cv) (%)	Change on Chance of a Profit > Zero (%)
46	.25	-50.00	-10.34	-35.10	-7.24	-5.40	5.49	-.21
47	.35	-30.00	-6.20	-9.93	-4.90	-3.48	2.89	0
48	.40	-20.00	-4.13	-5.30	-2.21	-2.40	1.80	0
49	.45	-10.00	-2.07	-5.30	-1.31	-1.25	.84	0
50	.55	10.00	2.07	1.99	3.92	1.32	-.75	0
51	.60	20.00	4.13	3.97	6.37	2.70	-1.38	0
52	.65	30.00	6.20	12.58	6.95	4.16	-1.94	0

TABLE 12-8

VARIATIONS IN MOST LIKELY PROFIT/BARREL - e(b)

Parameters

n = 20
p = .15
Q(5%) = 5; Q(95%) = 79; Q(max) = 140; Q(min)= 1; (in 10^6bbl)
e(a) = \$.50; e(c) = \$1.15
C_u = \$.15 million

NOTE: Profit (\$x10^6)

Case No.	Lowest Profit/bbl e(b) (\$)	Best Estimate of Discounted Profit - E(P) (mean)	Discounted Profit (median)	Discounted Profit with 90% Confidence Limits Min.	Max.	Standard Deviation (σ)	Coeff. of Variation (cv)	Chance of a Profit > Zero (%)
Base	.88	68.02	56.03	1.51	170.00	53.66	.7889	95.8
53	.62	60.74	49.24	1.29	155.45	48.77	.8030	95.8
54	.70	62.97	52.12	1.36	160.00	50.17	.7966	95.8
55	.79	65.50	54.50	1.48	166.67	51.85	.7917	95.8
56	.97	70.54	57.83	1.57	177.78	55.58	.7879	95.8
57	1.06	73.07	60.16	1.60	185.00	57.59	.7882	95.8
58	1.14	75.31	62.00	1.70	188.89	59.44	.7893	95.8

TABLE 12-8A

VARIATION FROM BASE DATA - PARAMETER e(b)

e(b) = $.88

Case No.	Most Likely Profit/bbl e(b) ($)	Variation of Parameter From Base (%)	Change in Best Estimate of Discounted Profit - E(P) (%)	Change in Discounted Profit with 90% Confidence Limits (%)		Change in Standard Deviation (σ) (%)	Change in Coeff. of Variation (cv) (%)	Change in Chance of a Profit > Zero (%)
				Min.	Max.			
53	.62	-29.55	-10.70	-14.57	-8.36	-9.11	1.79	0
54	.70	-20.45	-7.42	-9.93	-5.88	-6.50	.98	0
55	.79	-10.23	-3.70	-1.99	-1.96	-3.37	.35	0
56	.97	10.23	3.70	3.97	4.58	3.58	-.13	0
57	1.06	20.45	7.42	5.96	8.82	7.32	-.09	0
58	1.14	29.55	10.72	12.58	11.11	10.77	.05	0

TABLE 12-9

VARIATIONS IN HIGHEST PROFIT/BARREL - e(c)

Parameters

n = 20
p = .15
Q(5%) = 5; Q(95%) = 79; Q(max) = 140; Q(min) = 1; (in 10^6bbl)
e(a) = \$.50; e(b) = \$.88
C_u = \$.15 million

NOTE: Profit (\$x10^6)

Case No.	Lowest Profit/bbl e(c) (\$)	Best Estimate of Discounted Profit - E(P) (mean)	Discounted Profit (median)	Discounted Profit with 90% Confidence Limits Min.	Max.	Standard Deviation (σ)	Coeff. of Variation (cv)	Chance of a Profit > Zero (%)
Base	1.15	68.02	56.03	1.51	170.00	53.66	.7889	95.8
59	.88	60.54	50.54	1.33	152.86	47.32	.7815	95.8
60	.92	61.65	51.39	1.36	155.00	48.22	.7821	95.8
61	1.03	64.70	53.92	1.45	165.56	50.76	.7845	95.8
62	1.27	71.35	58.02	1.57	178.45	56.68	.7945	95.8
63	1.38	74.39	60.65	1.63	189.00	59.53	.8003	95.8
64	1.50	77.71	62.92	1.70	198.00	62.72	.8071	95.8
65	2.30	99.87	77.68	2.73	271.67	85.29	.8540	95.9

TABLE 12-9A

VARIATION FROM BASE DATA - PARAMETER e(c)

e(c) = $1.15

Case No.	Highest Profit/bbl e(c) ($)	Variation of Parameter From Base (%)	Change in Best Estimate of Discounted Profit - E(P) (%)	Change in Discounted Profit with 90% Confidence Limits (%) Min.	Max.	Change in Standard Deviation (σ) (%)	Change in Coeff. of Variation (cv) (%)	Change in Chance of a Profit > Zero (%)
59	.88	-23.48	-11.00	-11.92	-10.08	-11.82	-.94	0
60	.92	-20.00	-9.36	-9.93	-8.82	-10.14	-.86	0
61	1.03	-10.43	-4.88	-3.97	-2.61	-5.40	-.56	0
62	1.27	10.43	4.90	3.97	4.97	5.63	.71	0
63	1.38	20.00	9.36	7.95	11.18	10.94	1.45	0
64	1.50	30.43	14.25	12.58	16.47	16.88	2.31	0
65	2.30	100.00	46.82	80.79	59.81	58.95	8.25	.10

TABLE 12-10

VARIATIONS IN ITERATIONS & DISTRIBUTION

Parameters

$n = 20$
$p = .15$
$Q(5\%) = 5$; $Q(95\%) = 79$; $Q(max) = 140$; $Q(min) = 1$; (in 10^6bbl)
$e(a) = \$.50$; $e(b) = \$.88$; $e(c) = \$1.15$
$C_u = \$.15$ million

NOTE: Profit ($\$ \times 10^6$)

Case No.	Factor Studied	Best Estimate of Discounted Profit - E(P) (mean)	Discounted Profit (median)	Discounted Profit with 90% Confidence Limits Min.	Max.	Standard Deviation (σ)	Coeff. of Variation (cv)	Chance of a Profit > Zero (%)
Base	1000 Iterations	68.02	56.03	1.51	170.00	53.66	.7889	95.8
Base	10,000 Iterations	67.04	55.28	1.55	171.14	53.68	.8007	95.9
Base	Beta Distribution for Profit/bbl	67.98	55.29	0.84	173.08	56.01	.8239	95.5

Figure 12-1
Cumulative Probability - 'n' Parameter Set

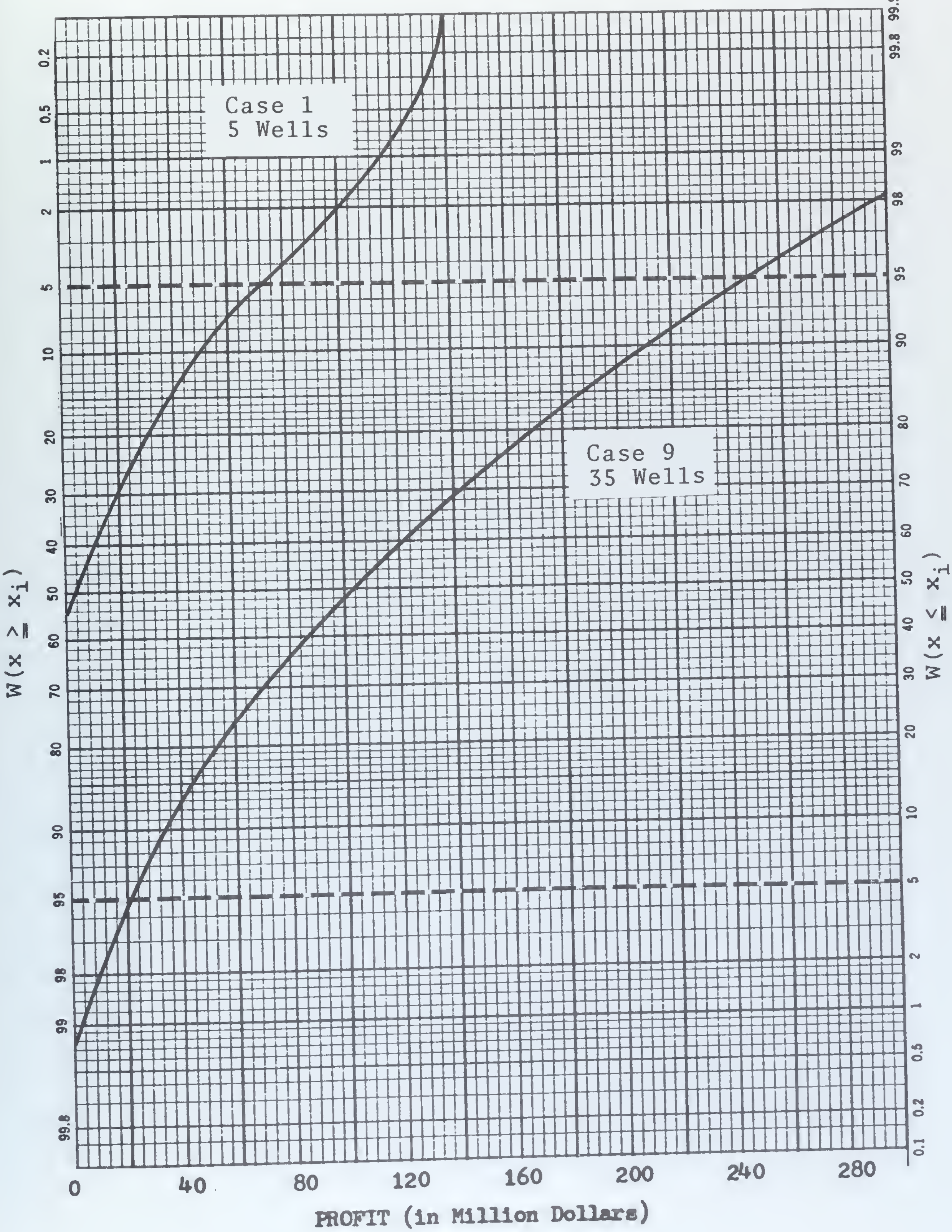


Figure 12-2
Cumulative Probability -'p'Parameter Set

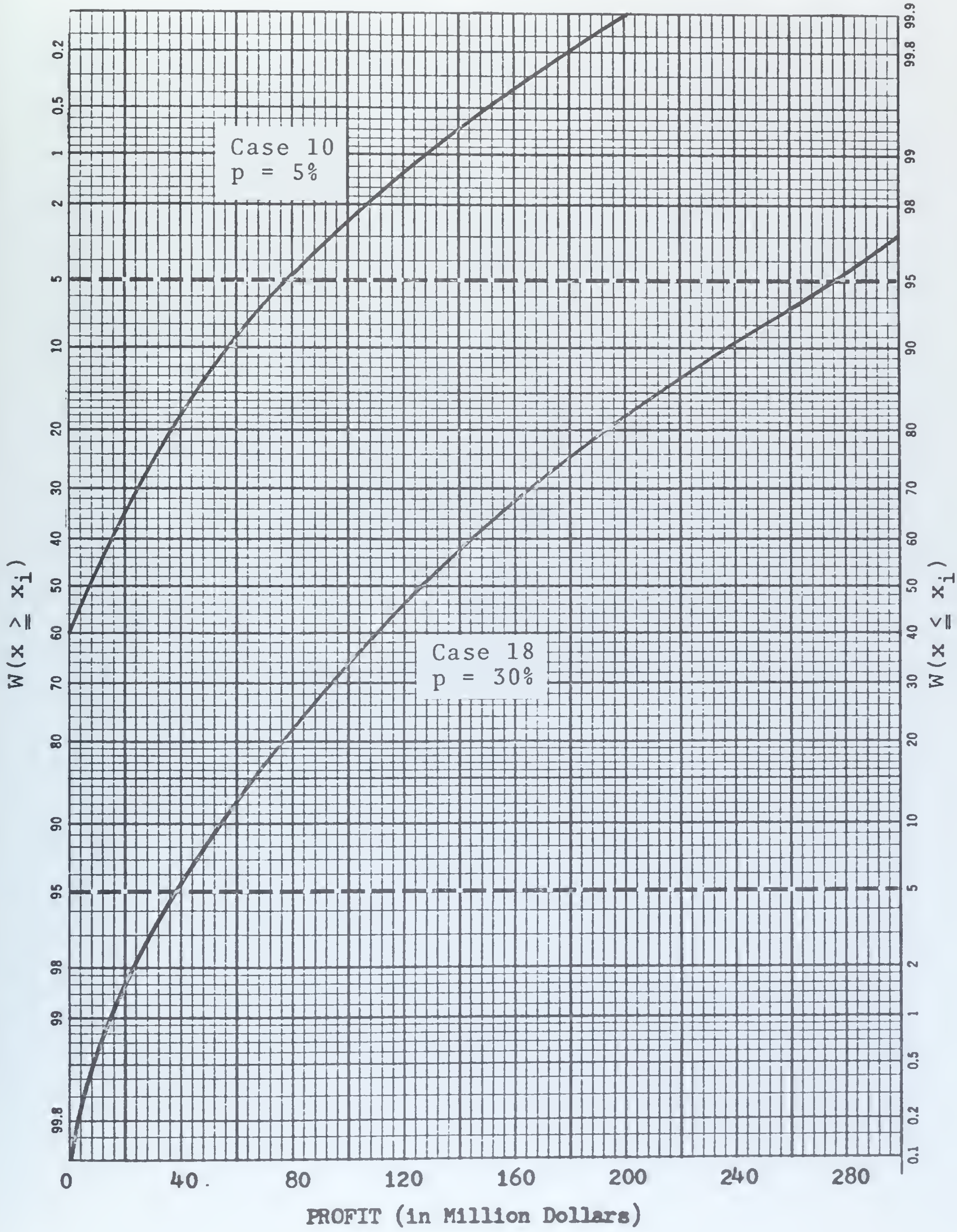


Figure 12-3
Cumulative Probability - Q at 5% Parameter Set

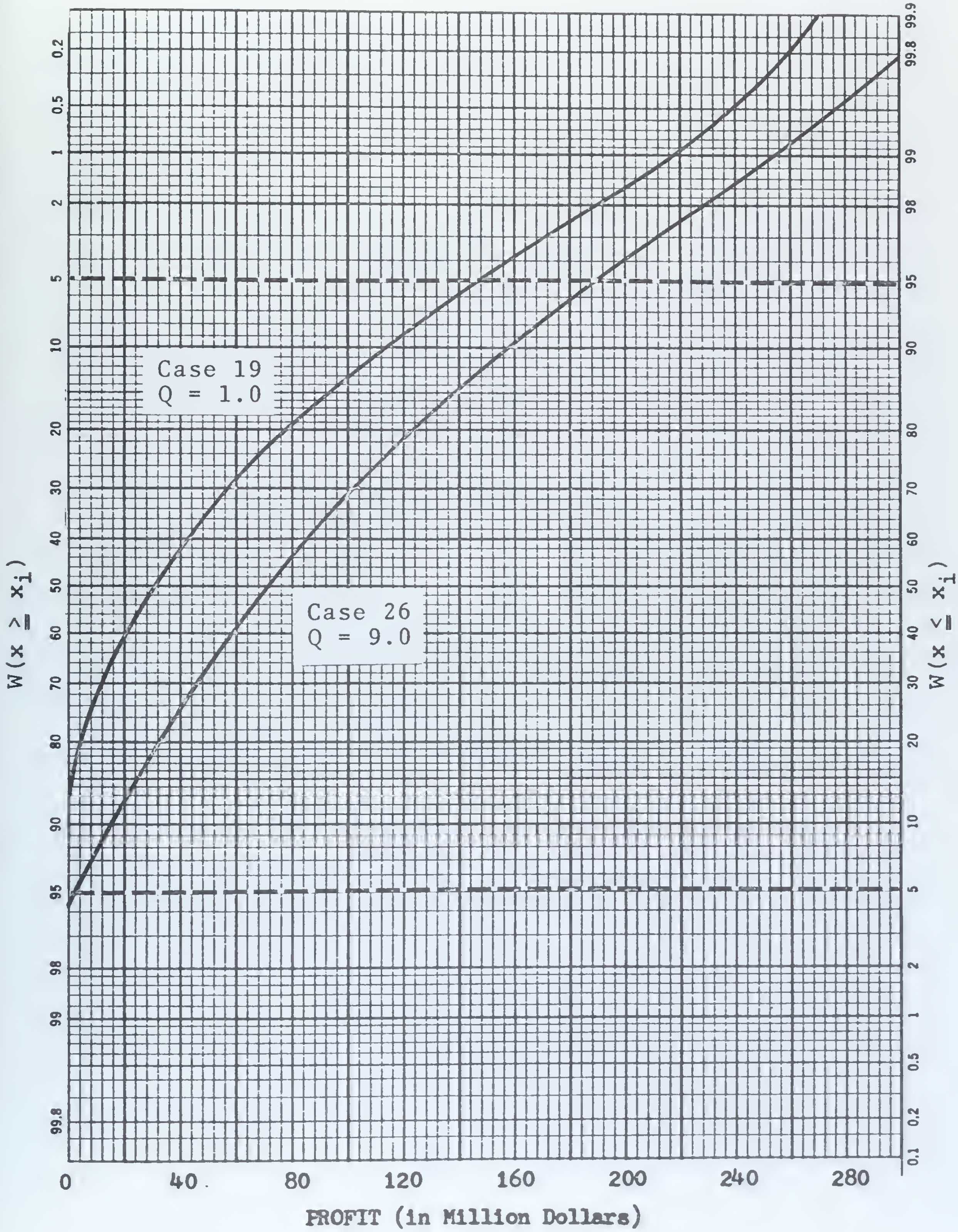


Figure 12-4
Cumulative Probability - Q at 95% Parameter Set

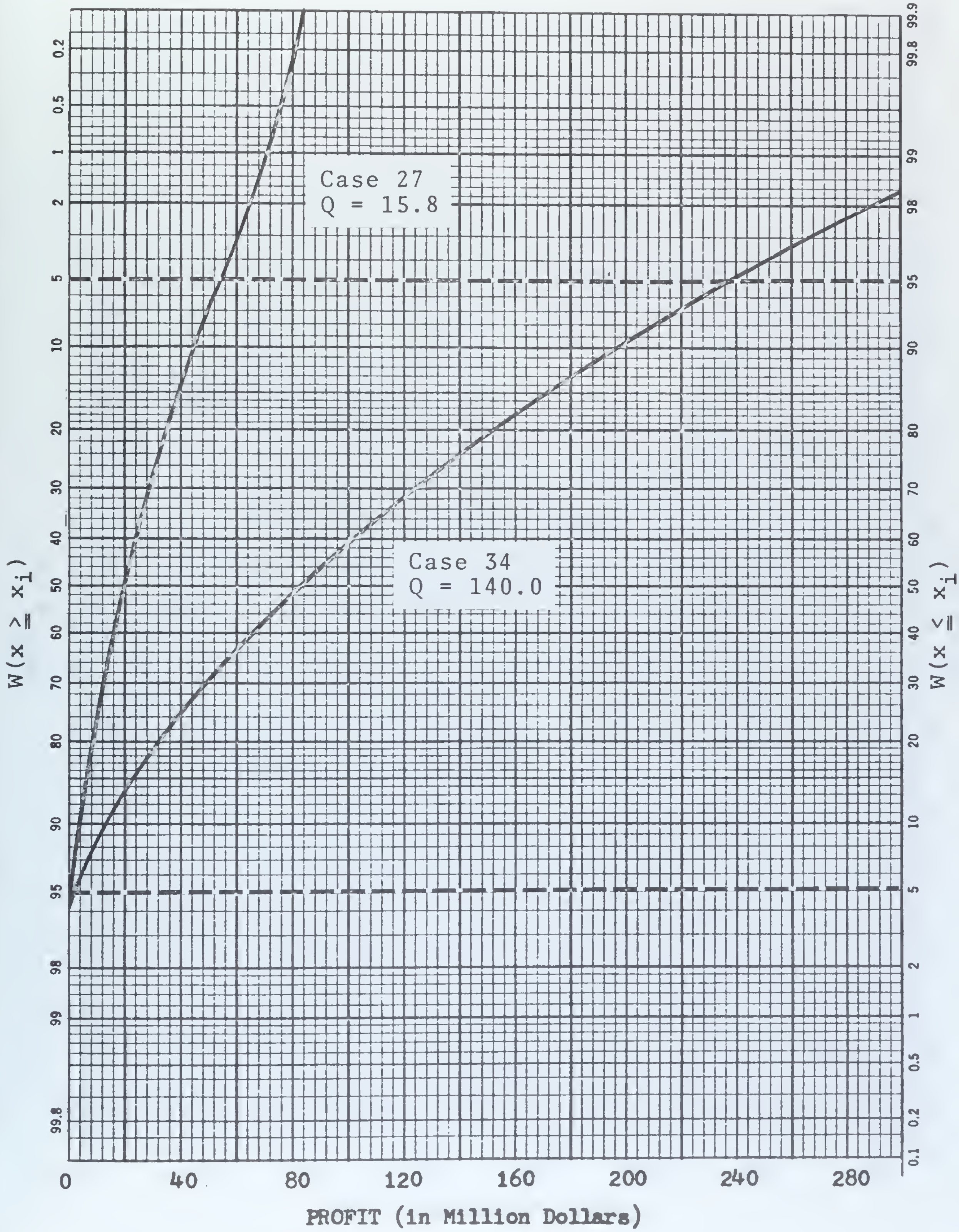


Figure 12-5
Cumulative Probability - C_u Parameter Set

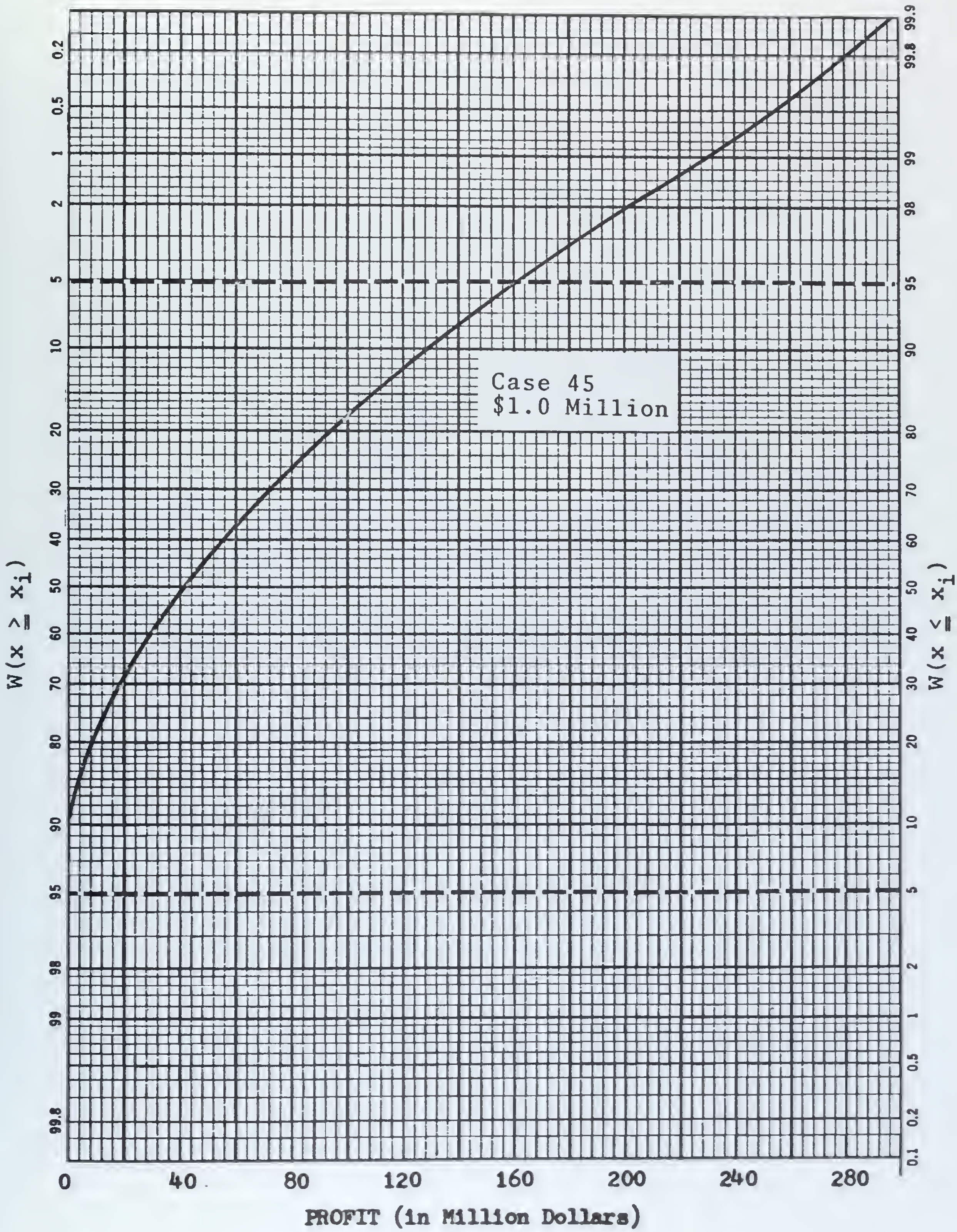


Figure 12-6
Cumulative Probability - e(a) Parameter Set

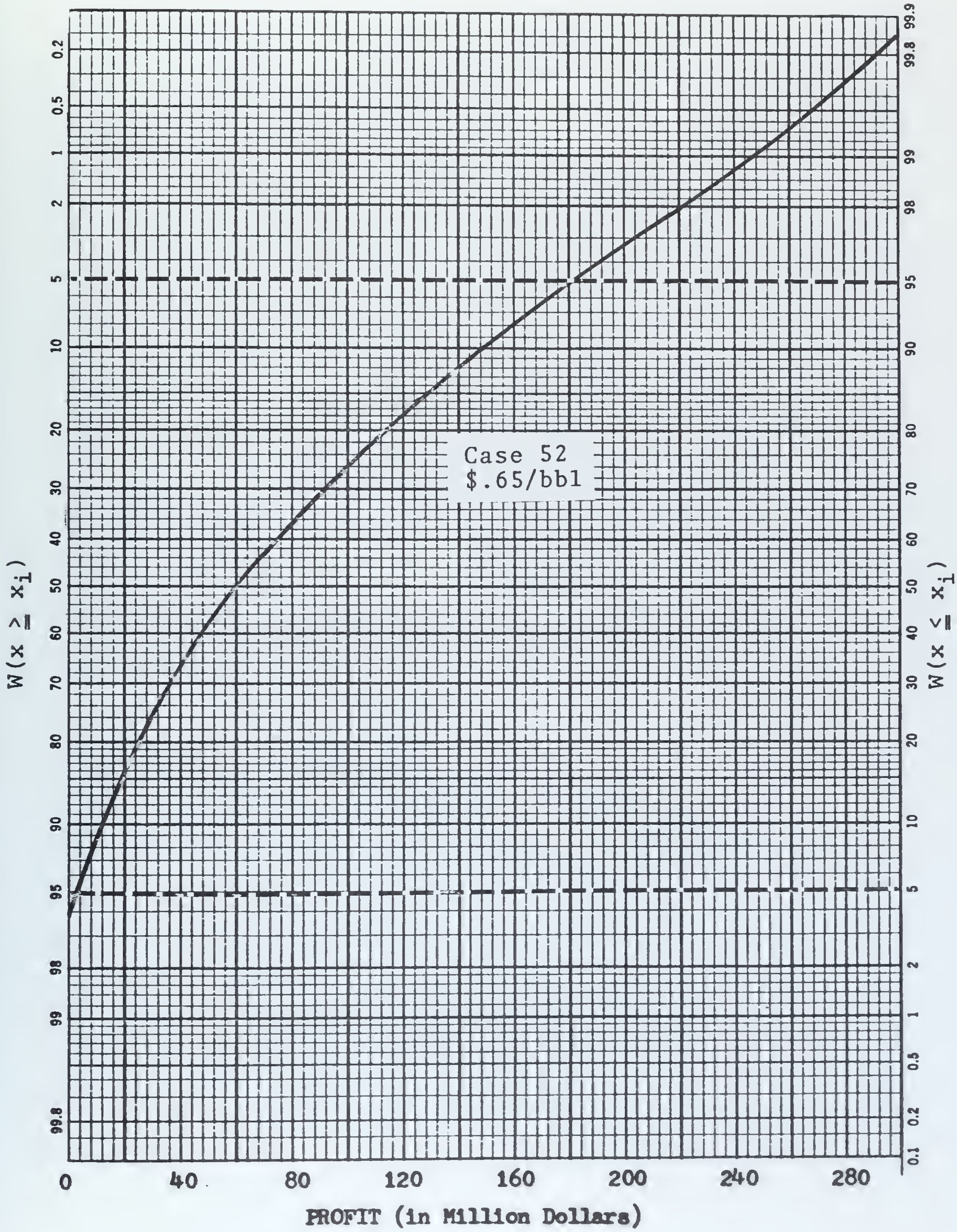


Figure 12-7
Cumulative Probability - e(b) Parameter Set

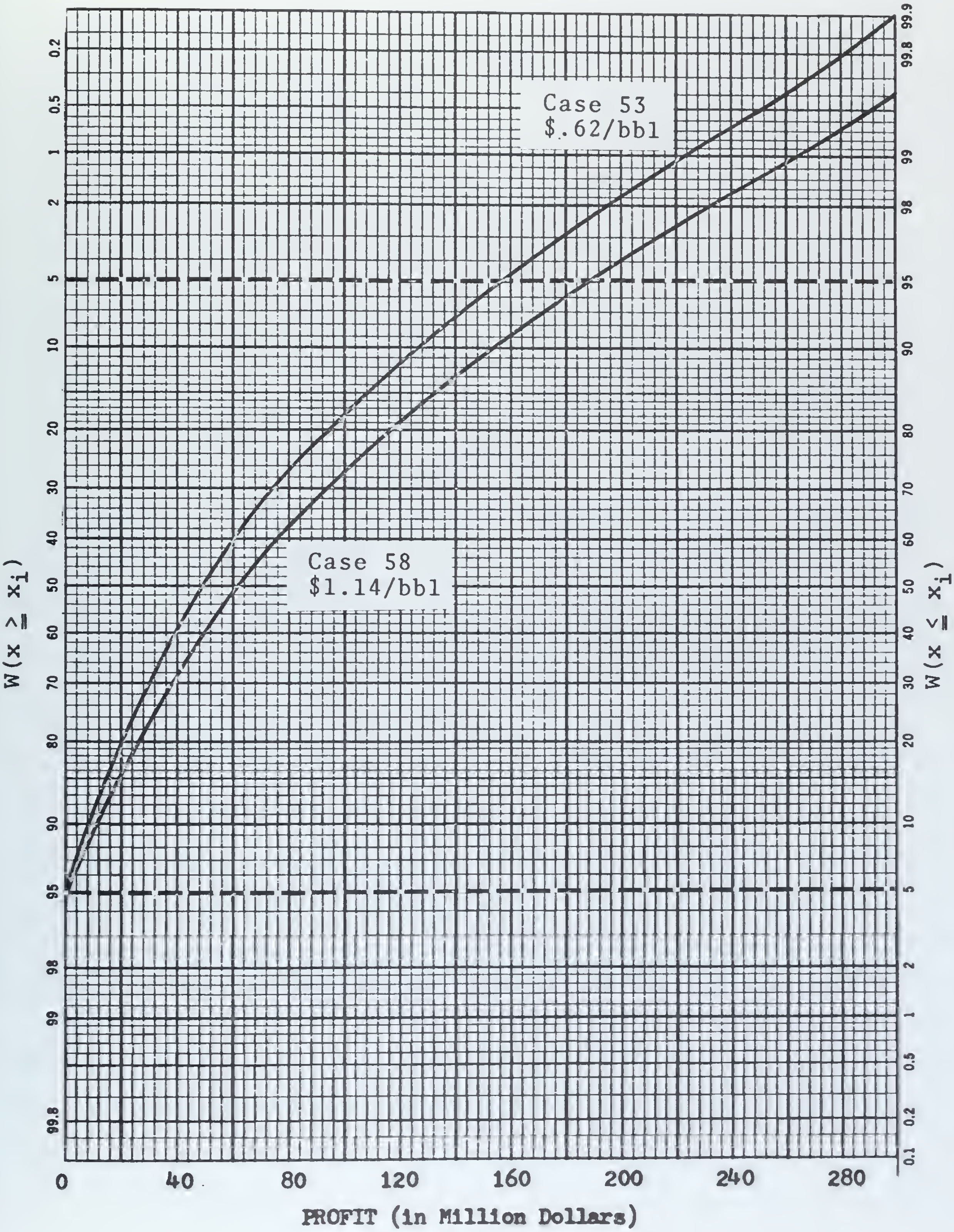
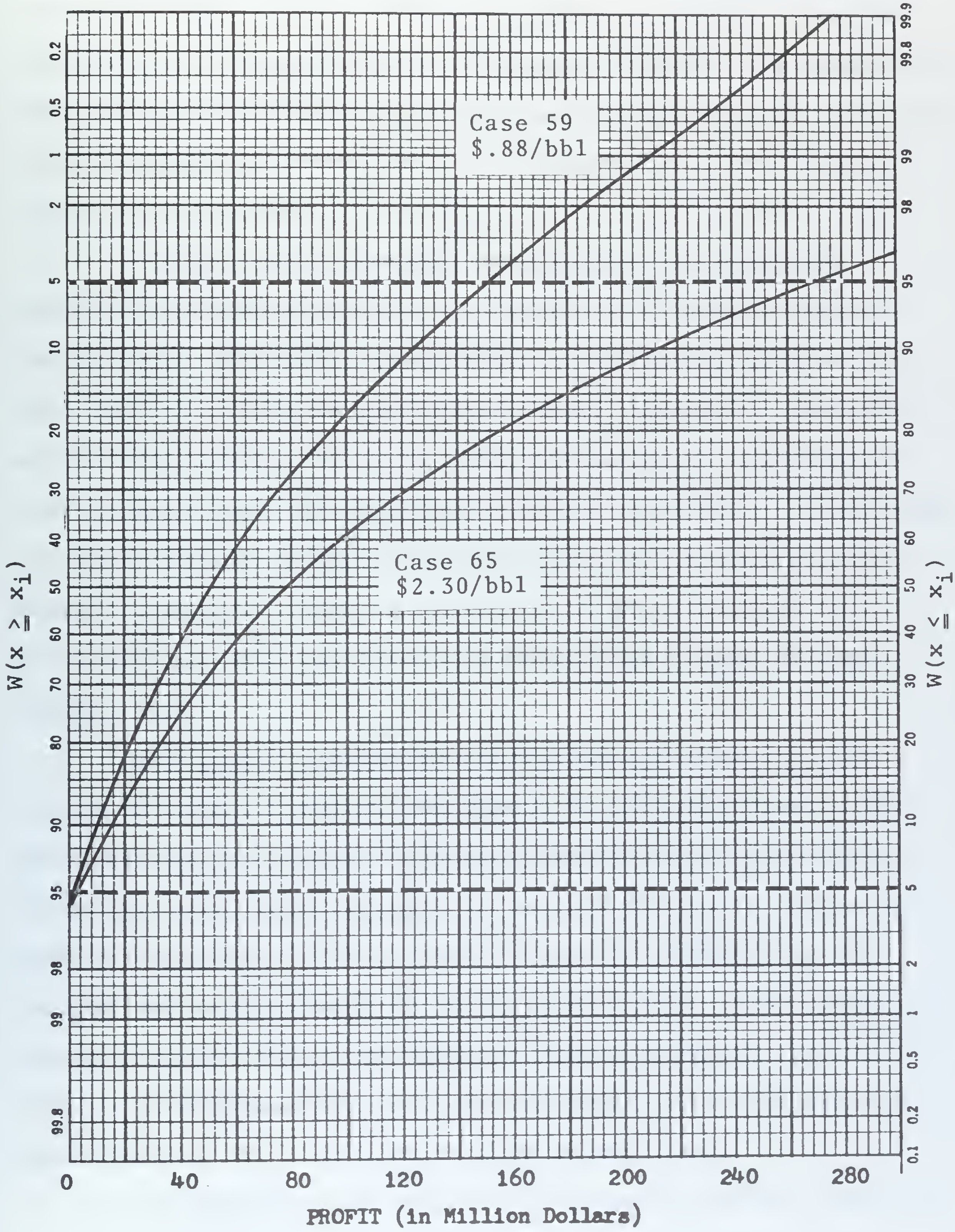


Figure 12-8
Cumulative Probability - e(c) Parameter Set



these parameter changes can be observed on Tables 12-1 through 12-9 and 12-1A through 12-9A and require no further datailed discussion. The analysis of a parameter change to a change in Best Estimate of Discounted Profit and to the risk as measured by the changes in Coefficient of Variation will be discussed in the next Section.

At this point however, some general observations regarding the sensitivity of the model to parameter changes can be made. First, an inspection or the Tables will reveal that the one single observed value which fluctuated the greatest on a percentage basis was the minimum value of discounted profit which could be expected with 90% confidence. The reason for this high percentage fluctuation was the relatively small numbers involved. Thus, a variation of $7.87 million on a $1.51 million Base resulted in a percentage change of approximately 980%.

The second observetion made from the Tables is that, except in isolated cases, the chance of a profit greater than zero was fairly constant at approximately 95 to 96%. Stating it another way, there appears to be only a 4 to 5% chance of a loss in a majority of the Cases. When the probability of success was reduced to 1 in 20 there was still a 62% chance of making a profit and, even when the number of wells drilled was reduced to 5 rather than 20, the probability of a profit was approximately 56% (the lowest value in any Case).

An inspection of Table 12-10 reveals that the substitu-

tion of the Beta distribution for discounted profit per barrel rather than the Triangular distribution has very little effect on the Best Estimate of Discounted Profit, E(P). The risk as measured by the Standard Deviation and the Coefficient of Variation is increased however, by about 4.4%. The minimum value of discounted profit with 90% Confidence Limits has been decreased by 4.34% and the maximum value has been increased by 1.78%. Additionally, the range of values with 90% Confidence Limits has been increased with the Beta distribution by 2.24%.

With regard to the Base data run at 10,000 iterations rather than 1000 iterations, it is seen from Table 12-10 that the value of E(P) only varies by 1.47% from the 1000 iteration value while the relative risk as measured by the Coefficient of Variation increases by 1.5% primarily due to the decrease in the absolute value of E(P).

12.3 Effects of Parameter Changes on the Best Estimate of Discounted Profit and Coefficient of Variation

The first question explored in this Section is the effect of a change in parameter value on the best estimate of discounted profit, E(P). This effect shall be assessed by calculating a Significance Ratio for each Case. The Significance Ratio is defined as the change in observed value (i.e. E(P)) to a change in the parameter value. In a conceptual sense the Significance Ratio measures the significance of a change in a parameter of the model on a value calculated by the model. We assume, of course, that all other parameters

(except the one being studied) remain the same. Mathematically, this can be expressed as:

$$SR = \left| \frac{\Delta \text{ Observed Value}}{\Delta \text{ Parameter}} \right| \tag{1}$$

With this Significance Ratio the relative importance of a change in various model parameters on the value being observed can be measured. For the profitability model it is possible, for example, to measure the relative importance of a change in dry hole costs, lowest profit per barrel, etc. on the best estimate of discounted profit for the total venture. For the analysis of the best estimate of discounted profit, Equation 1 can be rewritten:

$$SR = \left| \frac{\Delta E(P)}{\Delta \text{Parameter}} \right| \tag{2}$$

The values to be inserted in Equation 2 are obtained from Tables 12-1A through 12-9A. As an example, from Table 12-1A the following Significance Ratio is obtained:

For a change in parameter of -75.00% (Case 1) from the Base we have a commenserate reduction of E(P) of -76.77%. Inserting these values in Equation 2 we have:

$$SR = \left| \frac{-76.77}{-75.00} \right|$$

$$SR = 1.02$$

For Case 2 we have the following relationship:

$$SR = \left| \frac{-50.44}{-50.00} \right|$$

$$SR = 1.0.$$

In a similar fashion the Significance Ratios can be calculated for each of the nine Cases involving a variation of the number of wells drilled as shown on Table 12-1 and 12-1A. If this is done Significance Ratios ranging from .81 to 1.18 with a mean value of 0.99 are obtained.

Table 12-11 contains a summary of the Ranges and Mean Significance Ratios for each of the parameter sets contained on Tables 12-1 through 12-9 and Tables 12-1A through 12-9A. In addition, Table 12-11 contains a value of the Normalized Significance Ratio which reduces each Significance Ratio to a value on a 1-100 scale in proportion to its actual Significance Ratio. The Normalized Significance Ratio can be utilized to facilitate comparison of parameter sets

TABLE 12-11

SIGNIFICANCE RATIOS - E(P)

	Significance Ratios		
Parameter Set	Range	Mean	Normalized Mean
n	.81 - 1.18	.99	96
p	.96 - 1.10	1.03	100
Q at 5%	.19 - .40	.30	29
Q at 95%	.56 - .82	.72	70
Q(max)	.02 - .12	.07	07
Q(min)	0	0	0
C_u	0	.04	04
e(a)	0	.21	20
e(b)	0	.36	35
e(c)	0	.47	46

Several observations can be made from Table 12-11. First, it can be seen that a change in the number of wells drilled and probability of success are both almost equally significant as far as their effects on the value of E(P). In fact, there is virtually a 1 to 1 relationship between a change in the number of wells drilled and probability of success and a change in the best estimate of discounted profit. In other words, if the number of wells drilled is increased by 10%, almost a 10% increase in discounted profit can be expected. Likewise, if the probability of success is decreased by 20% about a 20% decrease in the discounted profit can be expected.

Also seen from Table 12-11 is that the next most significant parameter in the model is the estimated value of reserve size with 95% probability, Q at 95%. In fact, this parameter has a normalized significance of 70 which is approximately 2.5 times as critical as the lower estimate of the reserve size, Q at 5% probability, on the discounted profit. In a similar fashion it can be observed that variations in the values of the upper and lower boundary limits for reserve sizes have little or no significance on the discounted profit.

Although one might intuitively feel that changes in dry hole costs would have a material effect on the discounted profit it is seen that it only has a normalized significance of 4 which, in essence, is relatively insignificant in comparison to the other parameters.

In comparing the three parameters associated with the

discounted profit per barrel it is noted that, of the three, changes in the highest profit per barrel, e(c), are 2.3 times as significant as changes in the lowest value, e(a), and 1.3 times as significant as changes in the most likely value, e(b). Also observed is that of the 10 parameters measured, the value of e(c) ranks fourth in significance.

A similar analysis regarding risk can be made by computing the Significance Ratios for the Coefficient of Variation. In this case Equation 1 can be rewritten as:

$$SR = \left| \frac{\Delta \ CV}{\Delta \ parameter} \right| \tag{3}$$

Table 12-12 contains the results of these calculations.

TABLE 12-12

SIGNIFICANCE RATIOS - CV

	Significance Ratios		
Parameter Set	Range	Mean	Normalized Mean
n	.28 - 1.36	.67	100
p	.31 - 1.55	.64	96
Q at 5%	.04 - .43	.19	28
Q at 95%	.01 - .18	.06	9
Q(max)	.06 - .15	.10	15
Q(min)	0	0	0
C_u	.04 - .05	.04	6
e(a)	.06 - .11	.09	13
e(b)	0 - .06	.03	5
e(c)	.04 - .08	.06	9

From Table 12-12 certain observations regarding the significance of a change in model parameters on the relative risk of the venture can be made. First, it is observed that a change in the number of wells drilled and in the probability of success both have about the same effect on the risk of the venture. It is interesting to note that the Normalized Significance Ratios for E(P) are exactly reversed for CV. In this regard, Table 12-11 indicated that a change in Q at 95% had a greater significance than a change in Q at 5%. Table 12-12 reveals the opposite is true for the relative risk. Here a change in Q at 5% has a much greater significance than a change in Q at 95%. The same reversal is evident in the values of lowest and highest profit per barrel, e(a) and e(c).

It must be remembered, of course, that the Significance Ratio for risk (coefficient of Variation) does not indicate whether the risk is increasing or decreasing with a parameter change, but only the relative amounts that it is changing. For the increase or decrease correlation we must go to Tables 12-1 through 12-10 and 12-1A through 12-9A.

12.4 Convergence of the Expected Profit

If the Monte Carlo simulation model is executed only once, only one value for the discounted profit is generated and there is no distribution of values that can be analyzed. Since simulation is being used because the model is too complex for other types of analyses, there is no basis for relying on the single result as a typical output of the model - there is no

way to tell whether the result is an extreme case or about the average.

As the simulation process is repeated frequencies of occurrence are obtained from which the averages, the extremes, and what the shape of the distribution would be if the model had been executed infinitely many times can be inferred. In this regard, R.F. Barton [6] poses the following question:

> .... how reliable are the results from a limited number of executions? Two executions may give us a great deal more information than a single execution. One hundred iterations may give us hints of the extremes and shapes of the ultimate distributions. A thousand executions may perhaps give us results that approximate smooth curves. But a second thousand executions may suggest smooth curves of different locations and shapes. A third thousand executions may generate results that look even different. On the other hand, the separate sets of one thousand executions may look so similar we feel no need for further executions.

These possibilities raise a very practical question: How long to run a simulation model? One aspect of this question was analyzed by measuring the convergence of the discounted Expected Profit, E(P), averaged over the number of iterations run, with the discounted Expected Profit averaged over 1000 iterations.

As an example, if the value E(P) for the first iteration is $100.0 million and $50.0 million for the second iteration the discounted E(P) as determined by two iterations is $75.0 million. We can then compare this value with the E(P) averaged over 1000 iterations, say $60.0 million, to determine the rapidity of convergence. If we were satisfied with an Expected

Profit that varied no more than ± 5% from the value which would be obtained with 1000 iterations (90% certainty) then by plotting or analyzing the data we could determine how many iterations were necessary to reach this point. It may turn out, in our example, that a value of \$60.0 million ± \$3.0 million was achieved after only 500 iterations. By performing this analysis with many different values of model variables, using the same random numbers, and each time determining the point beyond which the averaged E(P) varies by no more than ± 5% of the 1000 iteration value, we may be able to infer the number of iterations necessary for 90% confidence limits for this particular model.

This is, in fact, what has been done. For each set of model variables the iteration beyond which the value of E(P) does not vary by more than ± 5% from the value of discounted expected profit after 1000 iterations was measured. These values are contained on Table 12-13. Figures 12-9 through 12-14 graphically depict this convergence process for various combinations of model parameters.

In addition to this analysis, the model was run with the base data for a total of 10,000 continuous iterations to determine if any periodicities in the variables, as suggested by Barton [7], occur which would materially effect the value of the discounted Expected Profit after 10,000 iterations. Table 12-10 and Figure 12-15 contain the results of this test. It can be seen from Figure 12-15 that no periodicities occur

and the 10,000 iteration value of E(P) is, in fact, only slightly different from the 1000 iteration value.

A visual inspection of the data contained on Table 12-13 indicated that Expected Profit, E(P), does not converge with any regularity, either within 80% or 90% confidence limits. It is seen that even within a parameter set(i.e. number of wells drilled) the number of iterations vary greatly. The only significant fact which can be discerned from the data is that values of E(P) within 80% confidence limits can be achieved anywhere from 10 to 339 iterations and within 90% confidence limits anywhere from 63 to 710 iterations. If the highest and lowest iteration value is eliminated the range for 80% confidence limits is 18 to 242 iterations and the range for 90% confidence limits is 68 to 596 iterations.

TABLE 12-13

CONVERGENCE OF E(P) WITH NUMBER OF ITERATIONS WITHIN CONFIDENCE LIMITS

Case No.	Factor Being Varied	E(P) 1000 Iterations	Within 80% Conf. Limits (± 10%) No. of Iterations	Within 90% Conf. Limits (± 5%) No. of Iterations
Base	-	68.02	117	207
1	n	15.80	239	502
2	n	33.71	242	457
3	n	39.49	190	440
4	n	51.48	186	268
5	n	60.02	147	195
6	n	73.52	65	107
7	n	83.12	111	261
8	n	99.47	136	238
9	n	115.37	10	63
10	p	18.19	339	710
11	p	33.75	213	268
12	p	43.56	204	395
13	p	81.08	108	255
14	p	90.46	41	83
15	p	100.50	56	167
16	p	114.13	42	99
17	p	125.12	26	68
18	p	137.27	40	96
19	Q(5%)	46.15	120	543
20	Q(5%)	61.16	117	596
21	Q(5%)	63.24	117	206
22	Q(5%)	65.30	117	207
23	Q(5%)	69.34	17	129
24	Q(5%)	70.90	17	129
25	Q(5%)	72.89	17	129
26	Q(5%)	80.42	42	129
27	Q(95%)	23.45	105	129
28	Q(95%)	52.42	17	207
29	Q(95%)	57.50	17	128
30	Q(95%)	62.64	18	207
31	Q(95%)	72.63	117	129
32	Q(95%)	77.14	117	207
33	Q(95%)	81.69	118	207
34	Q(95%)	97.46	119	473

TABLE 12-13 Continued

Case No.	Factor Being Varied	E(P) 1000 Iterations	Within 80% Conf. Limits (± 10%) No. of Iterations	Within 90% Conf. Limits (± 5%) No. of Iterations
35	Q max	64.35	43	129
36	Q max	69.29	18	305
37	Q min	68.02	117	207
38	Q min	68.02	117	207
39	Q min	68.18	22	129
40	Cu	69.30	18	129
41	Cu	68.87	18	129
42	Cu	67.17	117	207
43	Cu	65.47	118	207
44	Cu	62.07	117	265
45	Cu	53.58	119	539
46	e(a)	60.99	117	207
47	e(a)	63.80	117	207
48	e(a)	65.21	117	129
49	e(a)	66.61	117	207
50	e(a)	69.43	18	207
51	e(a)	70.83	19	129
52	e(a)	72.24	18	129
53	e(b)	60.74	117	129
54	e(b)	62.97	22	129
55	e(b)	65.50	22	129
56	e(b)	70.54	117	207
57	e(b)	73.07	117	207
58	e(b)	75.31	117	126
59	e(c)	60.54	117	207
60	e(c)	61.65	117	207
61	e(c)	64.70	117	207
62	e(c)	71.35	18	129
63	e(c)	74.39	22	129
64	e(c)	77.71	22	129
65	e(c)	99.87	22	129
Beta	Base	67.98	166	239

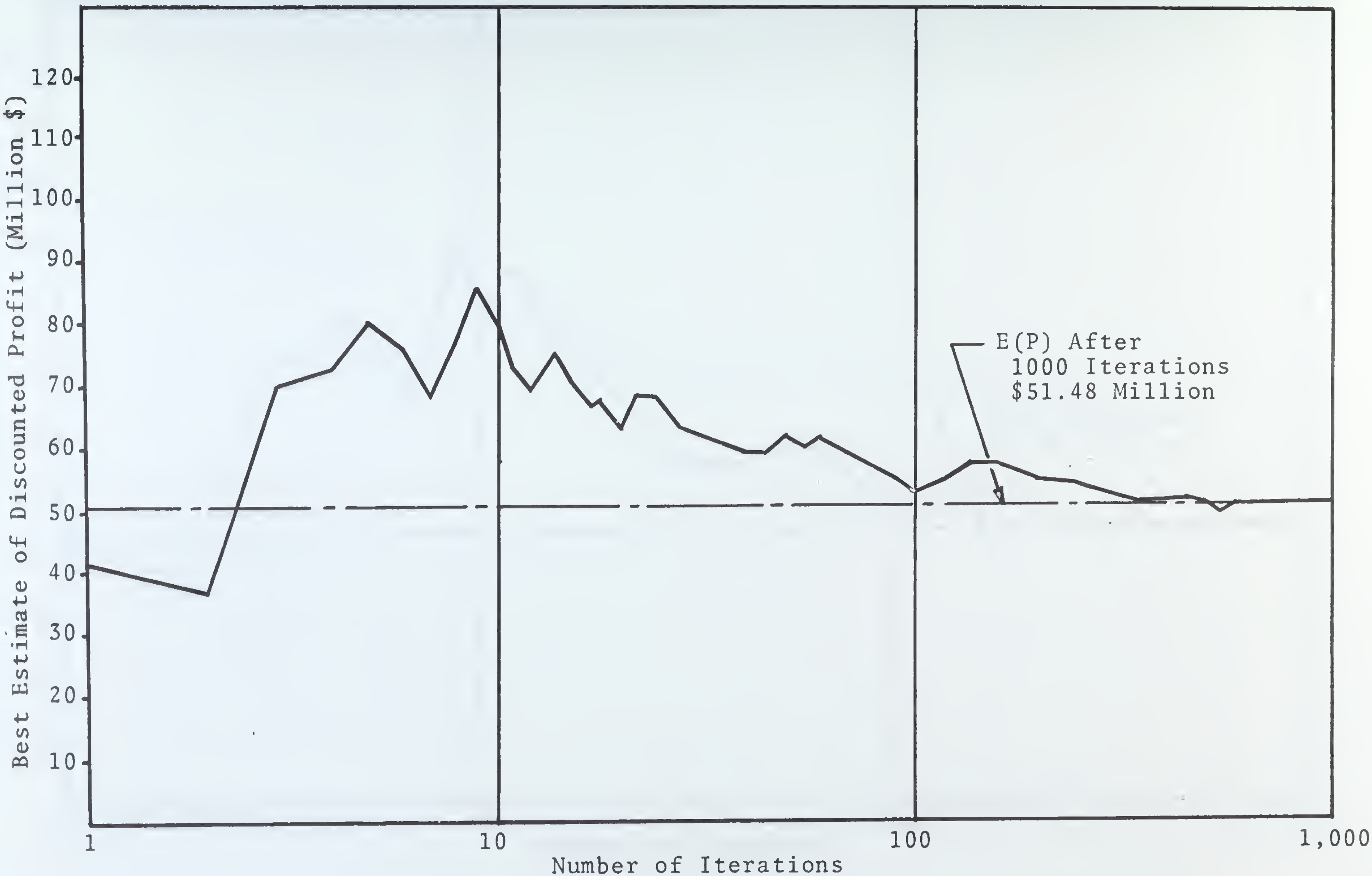


Figure 12-9 Convergence of E(P) - Case 4; n = 15

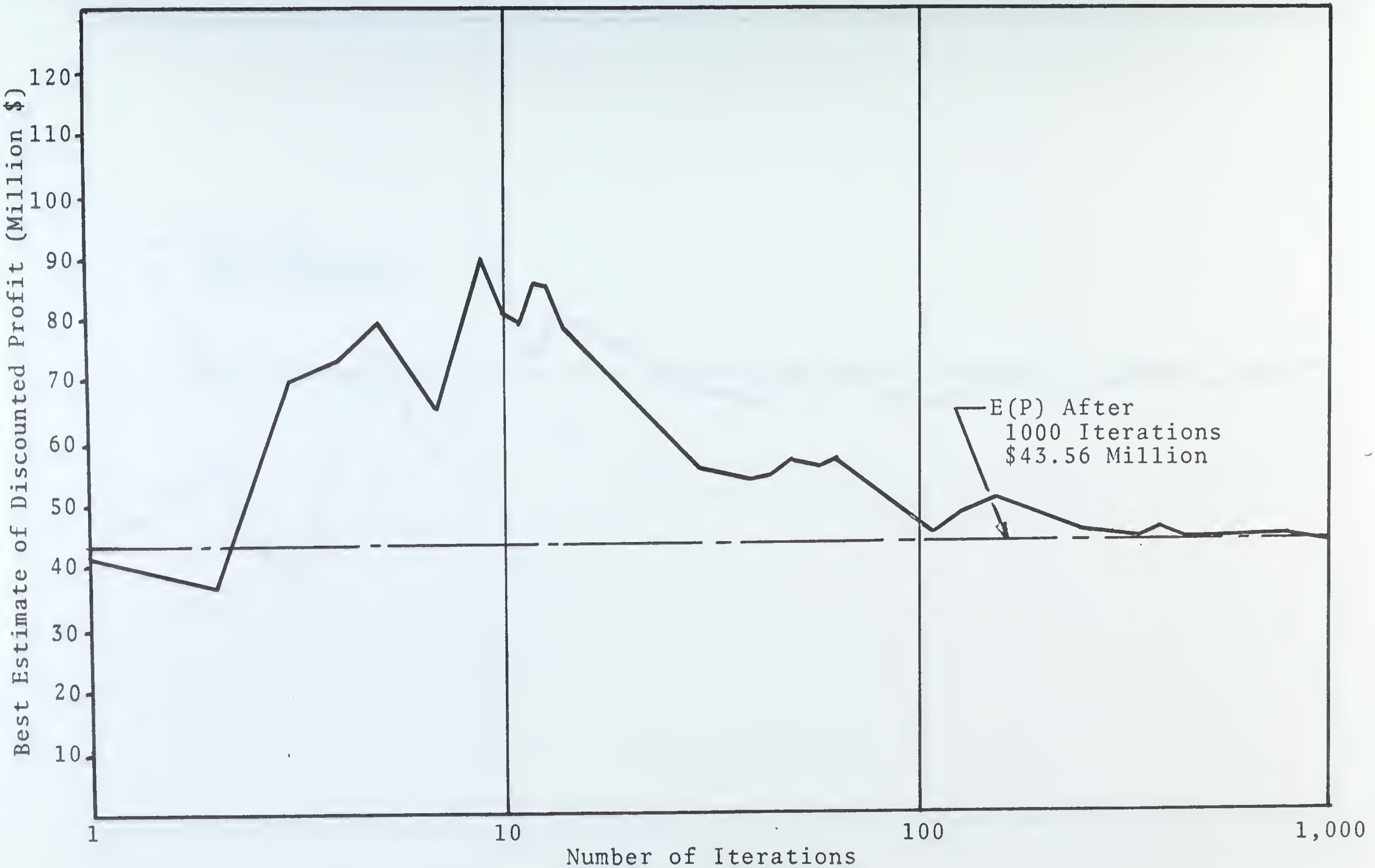


Figure 12-10 Convergence of E(P) - Case 12; p = 10%

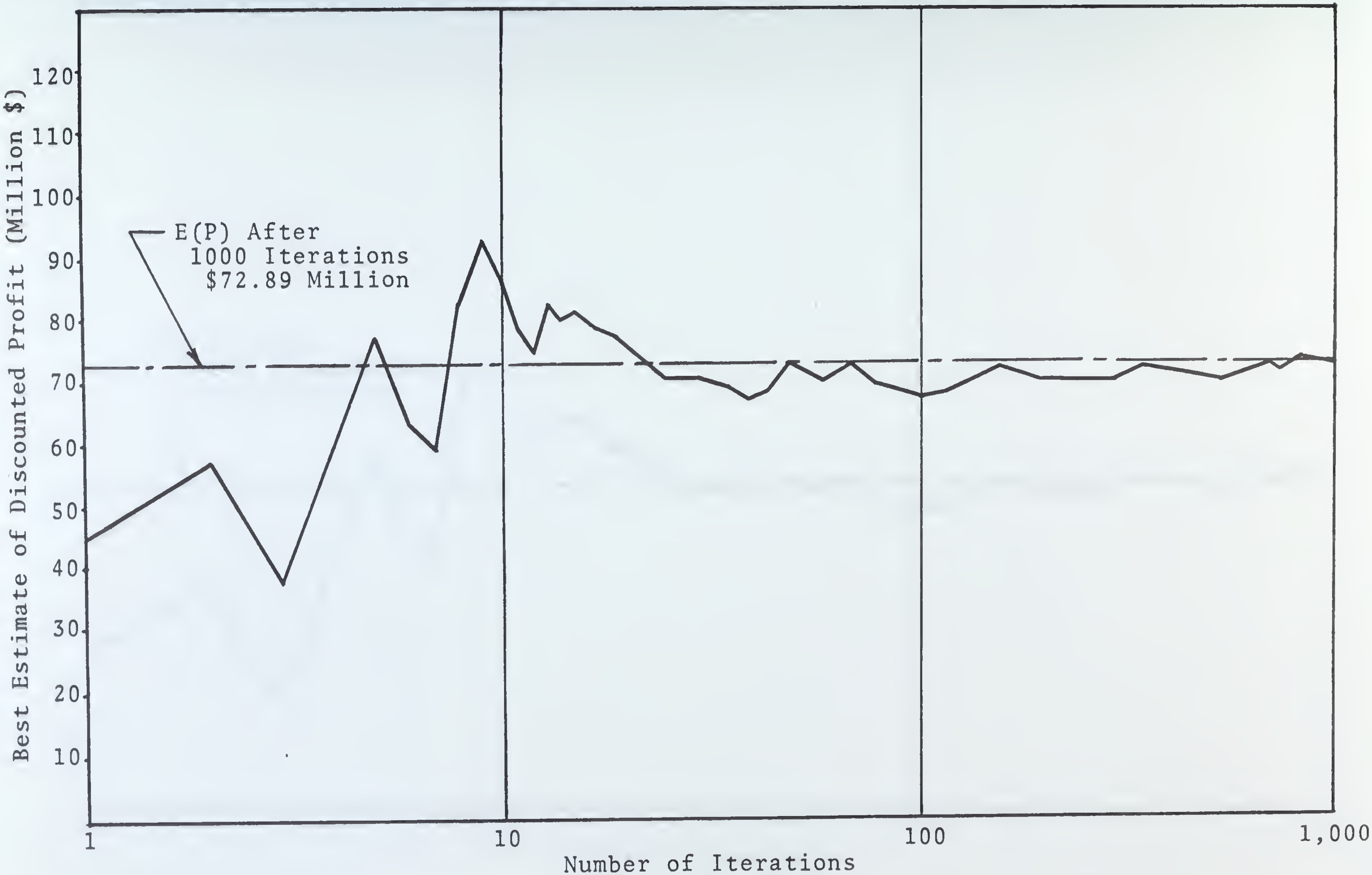


Figure 12-11 Convergence of E(P) - Case 25; Q at 5% = 6.5

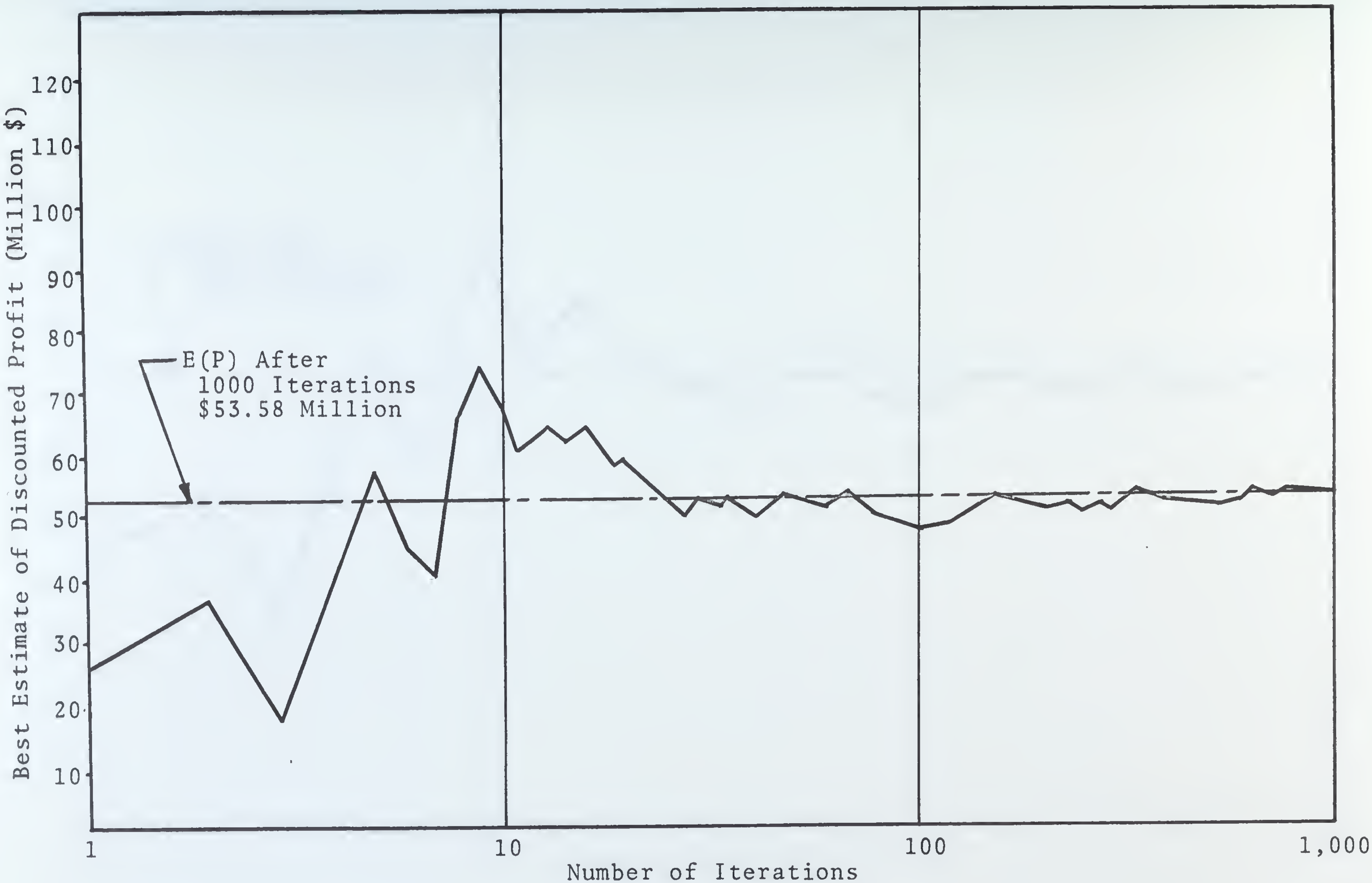


Figure 12-12 Convergence of E(P) - Case 45; C_u = \$1.0 Million

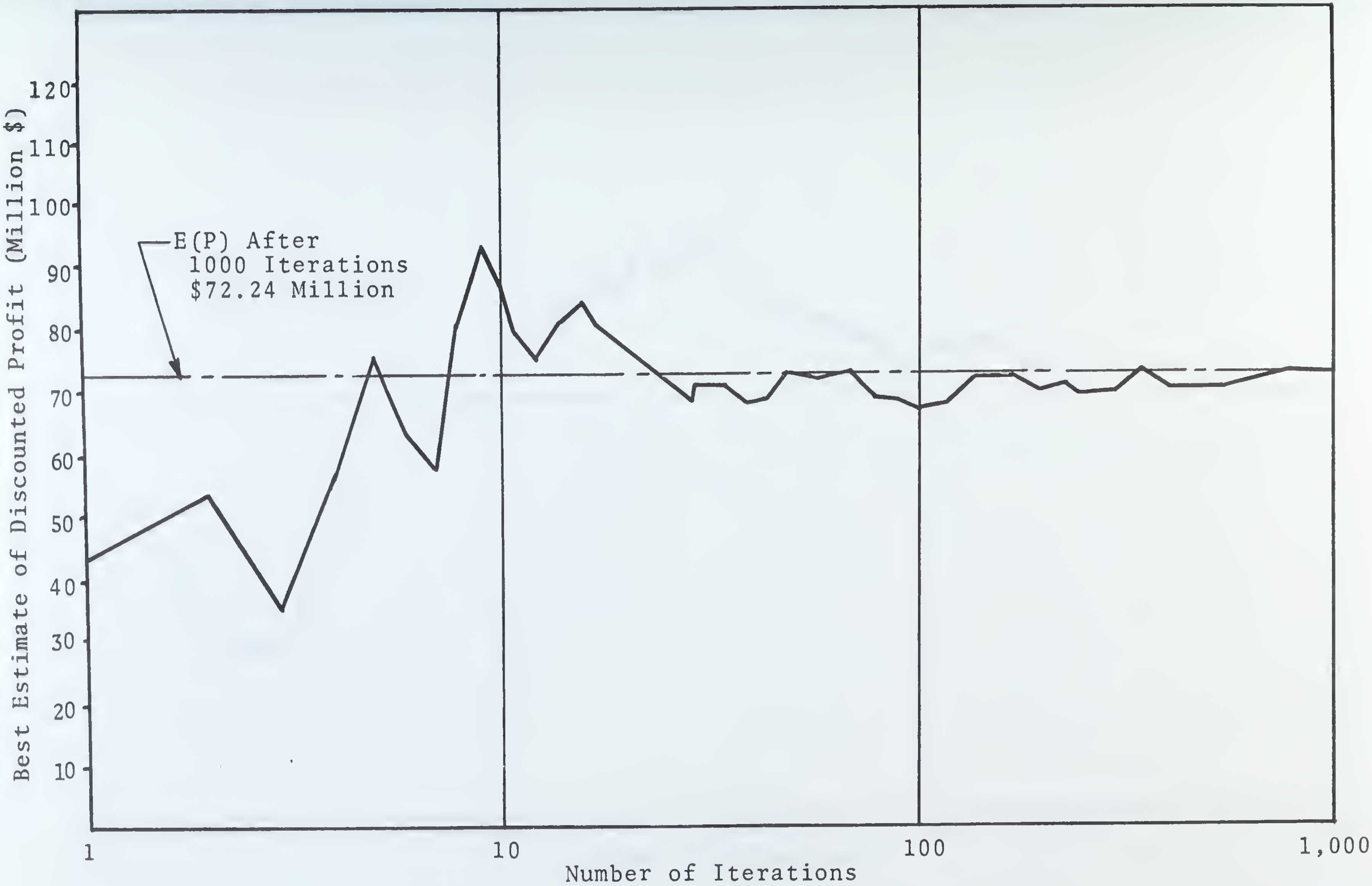


Figure 12-13 Convergence of E(P) - Case 52; e(a) = $.65

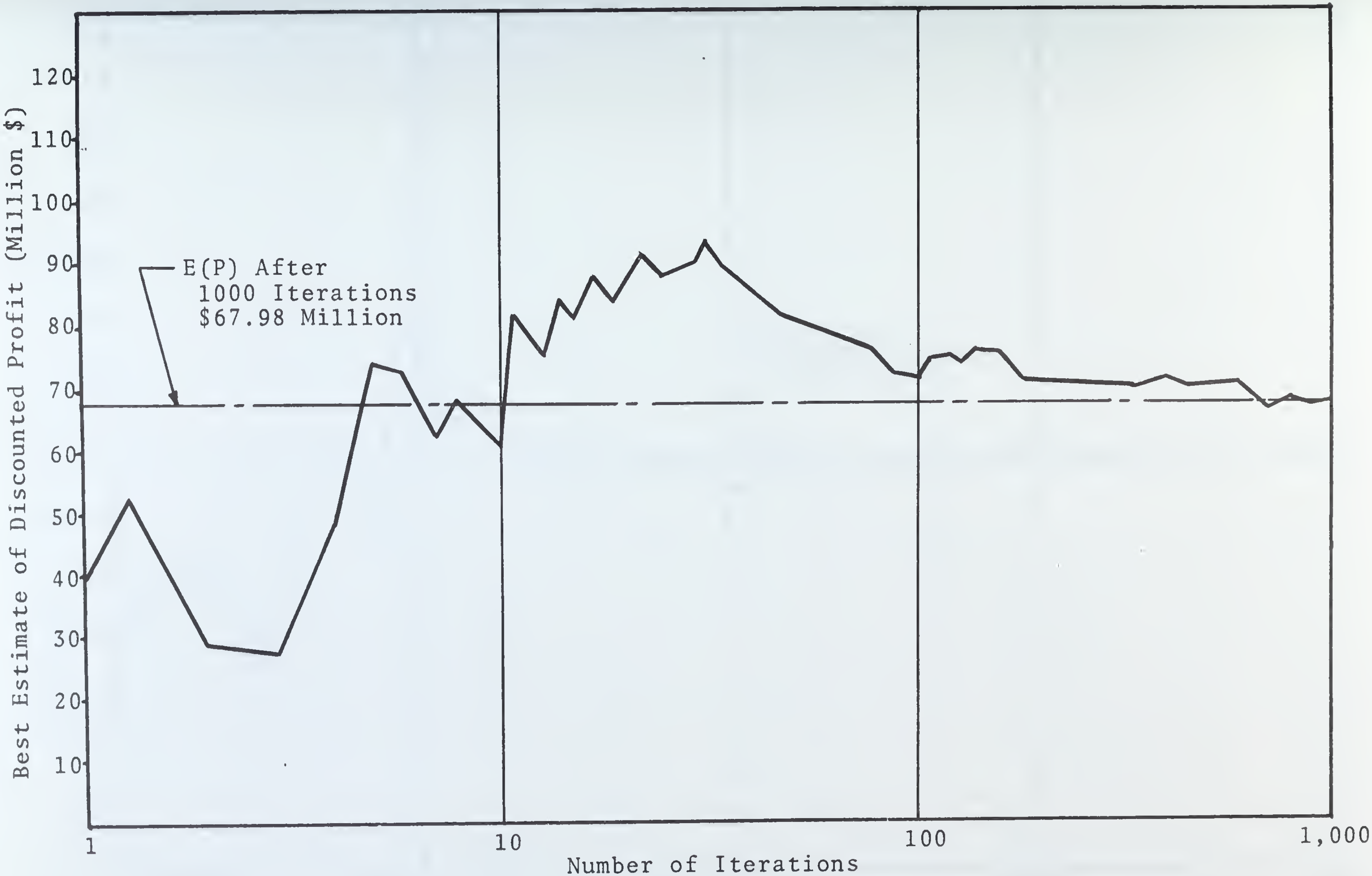


Figure 12-14 Convergence of E(P) - Beta Distribution

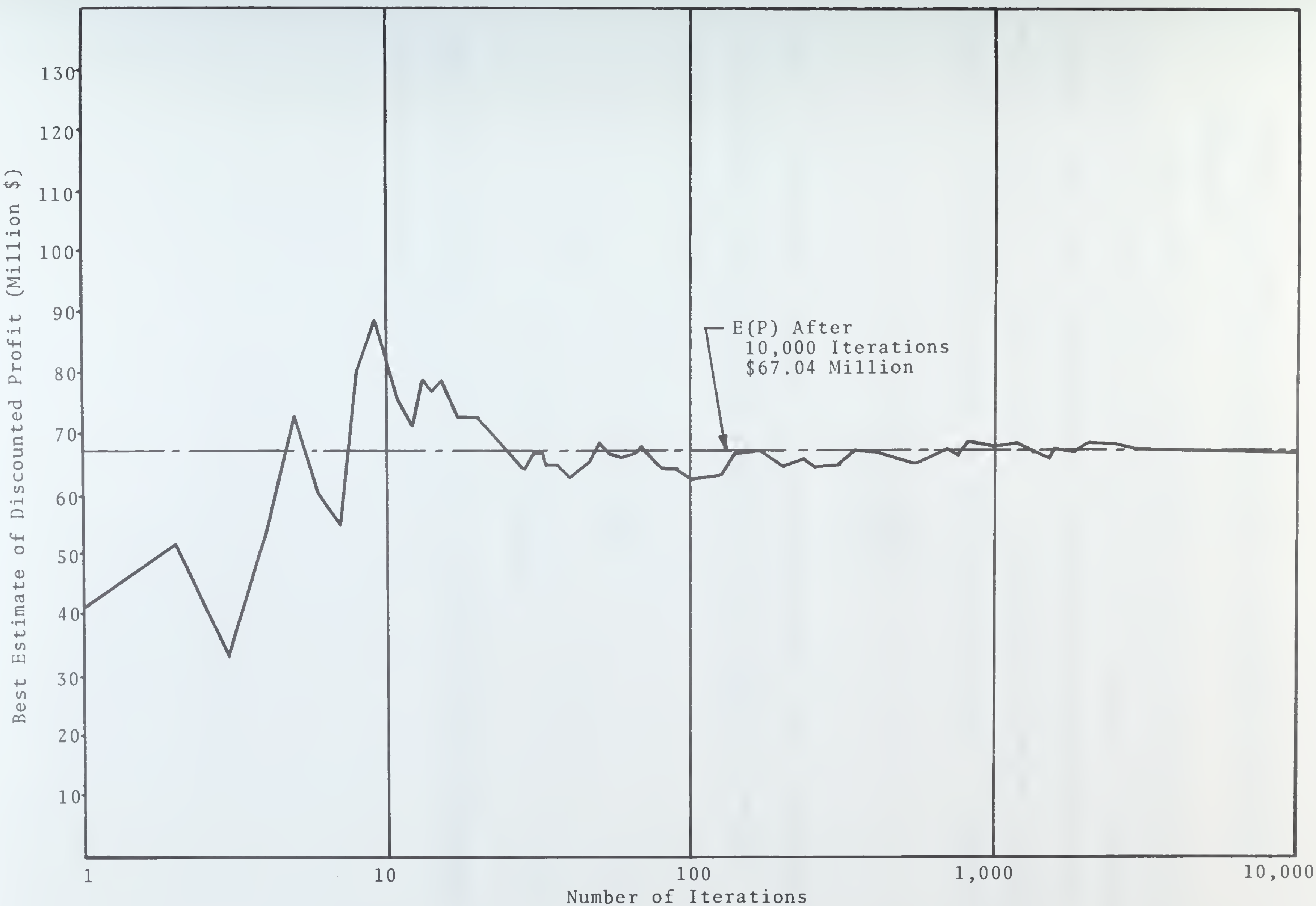


Figure 12-15 Convergence of E(P) - Base, 10,000 Iterations

12.5 Changes in Return and Risk with the Number of Wells Drilled

Since the risk of the venture has been measured by the coefficient of the variation it may be of interest to ascertain whether there is a mathematical relationship between the risk and the number of wells drilled. To determine whether a mathematical relationship exists we start with Equation 7, Chapter V:

$$CV = \frac{1}{\sqrt{n}} \sqrt{\frac{1-p}{p}}$$

Now, if p is held constant it can be seen that the Coefficient of Variation should be proportional to the number of wells drilled as follows:

$$CV \sim \frac{1}{\sqrt{n}} \qquad (4)$$

or,

$$CV \times \sqrt{n} = \text{Constant} \qquad (5)$$

Table 12-14 contains a test of this proportionality hypothesis using the data contained on Table 12-1

TABLE 12-14

PROPORTIONALITY TEST - CV to n

n	$\sqrt{n}$	Coeff. of Variation	Constant
5	2.23	1.59	3.54
10	3.16	1.11	3.52
12	3.46	1.07	3.70
15	3.87	0.92	3.56
18	4.24	0.84	3.56
20	4.47	0.79	3.53
22	4.69	0.75	3.52
25	5.00	0.71	3.55
30	5.48	0.64	3.50
35	5.92	0.62	3.67

It can be seen from Table 12-14 that the Coefficient of Variation is indeed proportional to $1/\sqrt{n}$. Thus, if the number of wells is increased by 4 times the risk will be reduced by a factor of 2.

It is also interesting to note that the Best Estimate of Discounted Profit per well, or E(P)/n, remains fairly constant while the risk, as measured by the Coefficient of Variation, decreases with $1/\sqrt{n}$. This can be seen on Table 12-15. This would tend to indicate that while the best estimate of return per well remains fairly constant with the number of wells drilled the risk associated with that return deminishes

as we increase the number drilled.

TABLE 12-15

BEST ESTIMATE OF PROFIT PER WELL AND RISK

Case	Number of Wells Drilled (n)	Best Estimate of Discounted Profit- E(P)	$\frac{E(P)}{n}$	Coeff. of Variation CV	Constant $\frac{1}{\sqrt{n}}$
1	5	15.80	3.16	1.59	3.54
2	10	33.71	3.37	1.11	3.52
3	12	39.49	3.29	1.07	3.70
4	15	51.48	3.43	0.92	3.56
5	18	60.02	3.33	0.84	3.56
Base	20	68.02	3.40	0.79	3.53
6	22	73.52	3.34	0.75	3.52
7	25	83.12	3.32	0.71	3.55
8	30	99.47	3.32	0.64	3.50
9	35	115.37	3.29	0.62	3.67

CHAPTER XIII

CONCLUSIONS

13.1 General

The following general conclusions are drawn from the results of this study:

1. The model developed herein provides a suitable vehicle for measuring the sensitivity of the parameters affecting the profitability of an exploratory oil drilling venture.

2. The parameters in the profitability model should include not only single best estimate values but, where appropriate, also a probability distribution of values.

3. The probability distribution best representing the number of successes is the Binomial distribution, especially when the probability of success and number of wells drilled are in the order of magnitude normally encountered in an exploratory drilling program. For probabilities of success approaching 50% and large numbers of wells drilled, the Normal or Gaussian distribution can closely approximate the Binomial distribution.

4. The reserve size discovered by a successful well is best represented by a Lognormal distribution. The values can be calculated manually using points scaled from a straight line drawn on log-probability paper between values of reserve size which can be expected with 5 and 95% probability. The lognormal distribution can be simulated in computer applications with excellent correlation using the Lognormally Distributed Variable Generator contained in this study.

5. The profit per barrel received over a period of years from oil produced must be discounted to reflect the time value of money.

6. The Triangular distribution function adequately serves to represent the discounted profit per barrel. The Beta distribution function provides only slightly different values of discounted profit per barrel when tested using the same input data used in the Triangular distribution. Both the Triangular and Beta distributions can be simulated in computer applications using the Generators contained herein.

7. The complexity of the distributions in the model precludes manual computation of profitability and sensitivity studies thereby necessitating the use of the Monte Carlo technique and a digital computer.

13.2 Sensitivity Studies

In the sensitivity studies, where individual parameters in the model were varied, with the remaining parameters being held constant, the following conclusions were drawn:

1. The one single observed value which fluctuated the greatest (on a percentage basis) was the minimum value of discounted profit within 90% Confidence Limits.

2. Except in isolated cases, the chance if a profit greater than zero in all tests remained fairly constant at 95 to 96%. Conversely, there remained a fairly constant chance of 4 to 5% of incurring a loss.

3. A subsitiution of the Beta distribution for discounted profit per barrel rather than the Triangular distribution makes very little effect on the best estimate of discounted profit for the venture or the risk as measured by the coefficient of variation.

4. There appears to be no periodicities in the parameters of the model and the best estimate of discounted profit calculated after 10,000 executions varies only slightly from the value calculated after 1,000 iterations.

5. It is possible to calculate values of the best estimate of discounted profit for the venture within 80% confidence limits (± 10% of the 1,000 iteration value of E(P)) by executing the model between 18 and 242 times; and within 90% confidence limits by executing the model between 68 and 596 iterations. Further, there appears to be no regular convergence of E(P) with the number of iterations performed.

6. A Significance Ratio, relating a change in observed value calculated by the model to a change in parameter value, provides an excellent tool for comparing the effects of changes of different model parameters, especially when the Significance Ratio is normalized on a 1 to 100 scale.

7. Using the Significance Ratios to relate parameter changes to discounted venture profitability, E(P) it is concluded that with the parameter values used:

a. Changes in probability of success and number of wells drilled have almost equal significance on changes in venture profitability.

b. Changes in the lower range of reserve size have only about 30% of the effect of changes in p or n on E(P). However, changes in the upper range have a 70% significance. The values selected at lower and upper boundaries for reserve sizes in the model have little or no effect on the value or E(P).

c. Changes in the highest value of discounted profit per barrel have more than twice the significance as changes in the lowest values of discounted profit per barrel; but even then, have only 46% of the significance as changes in n or p on E(P).

d. Changes in dry hole cost have relatively little effect on E(P).

8. Using the Significance Ratios to relate parameter changes to the relative risk of the venture as measured by the coefficient of variation, CV, it is concluded that with the parameter values used:

a. Changes in the number of wells drilled, n, and the probability of success, p, have about the same effect on the risk of the venture.

b. Changes in the lower range of reserve size are next in significance. Changes in this parameter are more than 3 times as significant as changes in the upper ranges of reserve sizes.

c. The value selected as the upper boundary for reserve size will effect the risk in the venture but the lower boundary limit will not.

d. Changes in dry hole costs do little to change the risk of the venture.

e. Changes in the lowest value of discounted profit per barrel is about 1.5 times as significant as changes in the highest value of discounted profit per barrel but even then, has only a significance of 13% as compared with the risk associated with changes in the number of wells drilled.

9. While the return per well remains fairly constant as the number of wells increase, the relative risk of the venture decreases as the square root of the number of wells drilled.

The seemingly large number of findings drawn from the results of this study are indicative of the value of this type of analysis. It is recognized, of course, that not all the conclusions listed herein will be considered of equal importance by the reader - it would be rare if they did. The fact that these findings could be drawn at all, however, leads to the overall conclusion that sensitivity analysis of this type provides a valuable tool for the decision-maker.

CHAPTER XIV

RECOMMENDATIONS

It is in the light of the problems facing most decision-makers, namely the "What if...?" questions cited in Chapter XI and the contention so lucidly expressed by William T. Morris [35] that at some point we must stop gathering data and refining our information and start recommending action, that the profitability model is ideally suited.

It is therefore recommended that a model of this type be used <u>not</u> to obtain absolute data on whether a profit will be made for the venture (or some other observation of this type) since other methods of analysis may be better suited for this purpose. Instead, it is recommended that the model be used to assess the relative sensitivity of the results to parameter changes in the model. With this knowledge of parameter sensitivity we can now devote time and effort to refining only those parameters which have a significant affect on the outcome.

For example, if we were concerned with the best estimate of discounted profit for the venture we would probably devote our efforts to refining our estimates of the probability of success, the upper range of reserve size and the highest value of discounted profit per barrel (assuming, of course, that the number of wells drilled is fixed). We would not, in this example, concern ourselves with such items as the dry hole costs, the lower range of reserve sizes and the like.

In this way, using the sensitivity analysis approach, the decision-maker can achieve the maximum return for the effort he can afford to expend.

APPENDIX A

THE LOGNORMALLY DISTRIBUTED VARIABLE GENERATOR

The FORTRAN IV Computer program for lognormally distributed deviates requires two different, uniformally distributed random fractions to generate one normal variable. With two uniformally distributed fractions, RA and RB, one generates a variable, V, which is normally distributed about a mean of 0, with a standard deviation of 1, thus,

$$V = (-2 \ln RA)^{.5} \times \text{COS } 2\pi \text{ (RB)}$$

To convert V to a variable QNORM, which is normally distributed about a mean of TMU, with a standard deviation SD, we simply use the relationship,

$$QNORM = (V)\ SD + TMU$$

By multiplying the natural logarithm base, e, to the normal deviate power, a lognormally distributed deviate, QLOGN, is generated.

$$QLOGN = e^{QNORM}$$

Once the Lognormally distributed deviate is generated it is tested against the lower and upper limils established as inputs to the program. If the deviate is less than the minimum value of QMIN, it is set to read QMIN. If it is greater than the maximum value of QMAX, it is set to read QMAX.

It will be noted that the program requires a value of

mean and standard deviation. These values can be obtained from the median value of Q at $W(\leq Q) = 50\%$, and the value of Q at $W(\leq Q) = 84\%$ (or, $y_m + \sigma_y$) as follows:

From Equation 3,

$$y_m \text{ (median)} = y_o \text{ (mean)}$$

From Equation 4B,

$$\ln x_m = y_o$$

$$Q_{median} = x_m$$

and,

$$TMU = y_o$$

then,

$$TMU = \log_e(Q_{median})$$

or

$$\underline{TMU = ALOG(QMEDIAN)}$$

$$y_o = y_m$$

$$y_m + \sigma_y = y_o + \sigma_y$$

$$y_m + \sigma_y = \ln Q \text{ at } W(\leq Q) = 84\%$$

or,

$$y_m + \sigma_y = \ln Q(84)$$

and,

$$\ln x_m = y_m$$

Therefore,

$$(y_m + \sigma_y) - y_m = \ln Q(84) - \ln(Q_m)$$

or,

$$\underline{SD = ALOG(Q84) - ALOG(QMEDIAN)}$$

The FORTRAN IV Computer program incorporating these concepts is listed as follows:

```
      PROGRAM QLOGN (INPUT,OUTPUT)
      READ 100, QMEDIAN, Q84, QMAX, QMIN
      SD = ALOG(Q84) - ALOG(QMEDIAN)
      TMU = ALOG(QMEDIAN)
      RA = RANF (0)
      RB = RANF (0)
      V = (-2.0*ALOG(RA)**.5* COS(2.0*3.1416*RB)
      QNORM = V*SD + TMU
      QLOGN = EXP(QNORM)
      IF (QLOGN - QMIN) 6, 7, 8
    6 QLOGN = QMIN
      GO TO 7
    8 IF (QLOGN - QMAX) 7, 7, 9
    9 QLOGN = QMAX
    7 CONTINUE
      PRINT 120, QLOGN
  100 FORMAT (10X,4F10.3)
  120 FORMAT (20X,F10.3)
      END
```

APPENDIX B

THE TRIANGULAR DISTRIBUTION VARIABLE GENERATOR

The FORTRAN IV Computer program for deviates with a Triangular distribution requires one uniformally distributed random function, RANF(O). This uniformally distributed random function will assume either the value of $W(x < x_i)$, - WXLTXI - or $W(x > x_i)$ - WXGTXI - depending on whether the calculated value of x_i(XI) is equal to or less than the inputed value of b.

The FORTRAN IV program incorporating these concepts is listed as follows:

```
      PROGRAM TRIA (INPUT,OUTPUT)

      THIS PROGRAM CALCULATES THE BEST ESTIMATE OF PROFIT/BBL
      ASSUMING A * TRIANGULAR * DISTRIBUTION OF PROBABILITY.

            A = MINIMUM PROFIT/BBL EXPECTED
            B = MOST LIKELY VALUE OF PROFIT/BBL EXPECTED
            C = MAXIMUM PROFIT/BBL EXPECTED

      READ 112, A, B, C,
      CALL RANF(O)
      WXLTXI = RANF(O)
      WXGTXI = 1.0 - WXLTXI
      XI = A + SQRT(WXLTXI*(B-A)*(C-A))
      IF (XI.LE.B) 55,50
   50 XI = C - SQRT(WXGTXI*(C-B)*C-A))
   55 PROFIT = XI/100.
      PRINT 300, PROFIT
  112 FORMAT (10X,3F10.2)
  300 FORMAT (15X,F4.2)
      END
```

APPENDIX C

THE BETA DISTRIBUTION VARIABLE GENERATOR

The FORTRAN IV Computer program for deviates with a Beta distribution is listed as follows:

```
      PROGRAM BETA (INPUT,OUTPUT)

      THIS PROGRAM CALCULATES THE BEST ESTIMATE OF PROFIT/BBL
      ASSUMING A * BETA * DISTRIBUTION OF PROBABILITY.

            A = MINIMUM PROFIT/BBL EXPECTED
            B = MOST LIKELY VALUE OF PROFIT/BBL EXPECTED
            C = MAXIMUM PROFIT/BBL EXPECTED

      READ 100, A, B, C
      XMU = (4.*B+A+C)/6.0
      XVAR = (C-A)**2/36.0
      BMEAN = (XMU-A)/(C-A)
      BVAR = XVAR/(C-A)**2
      XK1 = BMEAN*(BMEAN*(1.0-BMEAN)/BVAR-1.0)
      XK2 = XK1*((1.0-BMEAN)/BMEAN)
      CALL GAMMARN (XK1,GAM1)
      CALL GAMMARN (XK2,GAM2)
      BETA = (GAM1/(GAM1 + GAM2))*(C-A) +A
      PROFIT = BETA/100.
      PRINT 300, PROFIT
  100 FORMAT (10X,3F10.2)
  300 FORMAT (15X,F4.2)
      END

      SUBROUTINE GAMMARN(TK,GAM)
      GAMMA = 1.0
      K1 = TK
      TK1 = K1
      Y1 = RANF(0)
      IF (Y1-(TK-TK1)) 10, 10, 20
   10 K1 = K1 + 1
   20 DO 30 I = 1,K1
      Y2 = RANF(0)
   30 GAMMA = GAMMA*Y2
      GAM = -ALOG(GAMMA)
      RETURN
      END
```

APPENDIX D

COMPUTER PRINTOUT FOR BASE DATA

PROFIT DISTRIBUTION OF AN EXPLORATORY DRILLING PROGRAM

NO. OF WELLS DRILLED	PROB OF SUCCESS	DRY HOLE COST	QMEDIAN	Q84	QMAX	QMIN
20	.150	.150	20.00	46.00	140.00	1.00

THE TRIANGULAR DISTRIBUTION FOR DETERMINATION OF THE PROFIT PER BARREL HAS THE FOLLOWING EXTREMES IN CENTS -
A = 50.00 B = 88.00 C = 115.00

ITERATION	NUMBER OF SUCCESSES	FIELD SIZES	PROFIT PER BARREL	ITERATION PROFIT	BEST ESTIMATE OF PROFIT
1	2	52.90	.83	41.17	41.17
2	3	64.03	1.01	61.90	51.53
3	0	0.00	0.00	-3.00	33.36
4	4	114.52	1.02	114.13	53.55
5	5	189.57	.79	148.40	72.52
6	0	0.00	0.00	-3.00	59.93
7	3	34.12	.87	27.05	55.24
8	5	254.88	1.01	254.59	80.15
9	3	198.18	.81	157.93	88.80
10	2	32.61	1.01	30.36	82.95
11	0	0.00	0.00	-3.00	75.14
12	2	38.72	.93	33.17	71.64
13	5	179.08	.93	164.57	78.79
14	3	95.99	.57	51.88	76.87
15	3	166.86	.61	98.92	78.34
16	3	129.04	.65	81.67	78.55
17	2	28.20	.86	21.46	75.19
18	3	54.65	.80	41.00	73.29
19	3	73.73	.95	67.13	72.97
20	1	85.62	.97	80.58	73.35
21	5	127.41	.81	100.96	74.66
22	1	3.84	.92	.69	71.30
23	2	40.45	.79	29.10	69.46
24	4	59.44	.87	49.05	68.61
25	2	53.91	.81	41.17	67.52
26	3	49.27	.66	30.22	66.08
27	2	26.85	.70	16.03	64.23
28	5	96.30	.73	67.94	64.36
29	2	27.60	.74	17.80	62.75
30	4	187.40	.99	183.55	66.78
31	2	137.95	.64	85.37	67.38
32	4	83.15	.82	65.70	67.33
33	2	107.08	.82	84.63	67.85
34	0	0.00	0.00	-3.00	65.77
35	4	84.91	.85	69.90	65.89
36	2	38.40	.94	33.52	64.99
37	2	25.44	1.02	23.35	63.86
38	4	125.78	.73	89.62	64.54
39	1	6.76	.78	2.41	62.95
40	5	110.73	.89	95.83	63.77
41	0	0.00	0.00	-3.00	62.14
42	4	116.96	1.09	124.90	63.63

43	6	229.75	.91	206.55	66.96
44	1	32.97	.97	29.02	66.10
45	2	25.19	1.07	24.20	65.17
46	0	0.00	0.00	-3.00	63.68
47	2	181.65	1.02	183.29	66.23
48	3	84.76	.87	70.92	66.33
49	5	138.59	.59	79.57	66.60
50	8	160.84	1.04	165.66	68.58
51	4	91.12	1.04	92.52	69.05
52	1	50.01	.71	32.64	68.35
53	2	30.00	.67	17.45	67.39
54	1	43.41	.55	20.90	66.53
55	5	129.75	.94	119.35	67.49
56	1	19.93	.79	12.84	66.51
57	4	63.36	.92	55.84	66.32
58	3	72.45	.87	60.17	66.22
59	3	110.63	.63	66.80	66.23
60	3	100.93	.66	64.42	66.20
61	5	194.06	.94	181.04	68.08
62	3	170.74	.70	117.80	68.88
63	0	0.00	0.00	-3.00	67.74
64	3	74.02	.86	60.82	67.63
65	2	39.45	.87	31.43	67.08
66	4	89.92	.96	83.53	67.32
67	3	112.67	.84	91.91	67.69
68	6	172.22	.88	150.10	68.90
69	3	51.86	.99	48.86	68.61
70	2	47.19	.84	36.78	68.16
71	3	63.76	.91	55.78	67.98
72	2	76.24	.58	41.86	67.62
73	2	23.86	.80	16.45	66.92
74	3	110.39	.80	85.40	67.17
75	1	20.76	.84	14.53	66.47
76	3	95.34	.93	85.91	66.72
77	2	38.74	.87	30.88	66.26
78	4	23.97	.76	15.85	65.61
79	3	32.79	.79	23.31	65.08
80	3	50.87	.89	42.79	64.80
81	2	91.19	.73	64.26	64.79
82	5	96.51	.63	58.39	64.71
83	5	119.23	.91	106.21	65.21
84	0	0.00	0.00	-3.00	64.40
85	5	184.12	.73	132.41	65.20
86	2	31.18	.78	21.46	64.69
87	3	51.86	.86	41.95	64.43
88	3	80.81	.58	44.18	64.20
89	3	109.49	.95	101.59	64.62
90	3	55.49	.57	28.98	64.23
91	2	19.07	1.12	18.70	63.72
92	2	29.24	.94	24.86	63.30
93	3	88.87	.70	59.78	63.26
94	1	23.84	.88	18.05	62.78
95	3	83.49	.67	53.13	62.68
96	1	88.52	1.08	92.78	63.00
97	4	156.23	.84	129.02	63.68
98	0	0.00	0.00	-3.00	63.00
99	3	50.68	1.05	50.79	62.87
100	2	42.56	.93	37.08	62.61
101	0	0.00	0.00	-3.00	61.96
102	3	55.47	.87	45.67	61.81
103	4	88.88	.91	78.40	61.97
104	1	34.29	.89	27.59	61.64

105	5	124.19	.68	82.68	61.84
106	4	82.08	1.13	90.48	62.11
107	2	92.60	.65	57.52	62.06
108	4	141.82	.75	103.75	62.45
109	5	101.35	.87	85.45	62.66
110	3	45.26	.77	32.31	62.38
111	4	76.50	.77	56.34	62.33
112	2	27.12	.99	24.04	61.99
113	1	18.92	.96	15.36	61.58
114	7	83.56	.83	67.61	61.63
115	3	31.66	.84	24.00	61.30
116	2	85.46	.64	52.03	61.22
117	6	166.72	.84	137.93	61.88
118	5	199.84	.68	133.63	62.49
119	5	118.33	.94	108.42	62.87
120	3	150.97	.85	125.42	63.39
121	0	0.00	0.00	-3.00	62.84
122	4	256.55	.60	151.98	63.57
123	8	199.11	.93	184.32	64.56
124	2	111.75	.84	91.60	64.77
125	2	25.56	.80	17.70	64.40
126	0	0.00	0.00	-3.00	63.86
127	2	57.89	.73	39.42	63.67
128	6	83.49	1.03	83.96	63.83
129	5	245.33	.95	229.78	65.11
130	3	194.04	.95	181.68	66.01
131	5	108.37	1.02	108.45	66.34
132	3	72.57	.54	36.64	66.11
133	2	18.69	1.01	16.22	65.74
134	3	59.56	.86	48.53	65.61
135	3	200.74	.92	182.97	66.48
136	5	128.45	.95	119.53	66.87
137	1	3.87	.88	.55	66.38
138	6	231.08	.74	169.64	67.13
139	2	46.63	.67	28.36	66.85
140	1	31.68	.66	18.03	66.50
141	3	52.13	.88	43.28	66.34
142	4	100.82	1.06	104.06	66.60
143	2	27.43	1.10	27.35	66.33
144	0	0.00	0.00	-3.00	65.85
145	1	8.60	.79	3.96	65.42
146	5	246.11	.93	227.67	66.53
147	3	39.53	.71	25.60	66.25
148	5	209.61	.99	205.16	67.19
149	4	84.89	.66	53.98	67.10
150	3	33.16	.75	22.37	66.81
151	3	37.88	.85	29.71	66.56
152	1	5.58	.65	.80	66.13
153	7	266.54	.71	187.86	66.92
154	2	47.87	.74	32.89	66.70
155	4	30.16	.96	26.68	66.44
156	7	196.48	.79	152.41	67.00
157	4	161.99	1.00	159.09	67.58
158	4	89.22	.83	71.30	67.61
159	2	40.85	.79	29.58	67.37
160	3	135.26	.80	105.98	67.61
161	3	31.66	1.06	30.89	67.38
162	5	95.17	.91	84.64	67.49
163	4	79.70	.84	64.64	67.47
164	1	5.40	.99	2.50	67.07
165	3	90.85	.85	74.85	67.12
166	2	66.02	.73	45.18	66.99

167	3	84.28	1.05	86.29	67.10
168	3	87.96	.89	75.31	67.15
169	4	48.41	.96	44.18	67.02
170	4	47.52	.95	42.63	66.87
171	3	49.62	.80	37.04	66.70
172	4	99.74	.92	89.48	66.83
173	0	0.00	0.00	-3.00	66.43
174	4	57.47	.99	54.24	66.36
175	4	163.81	.75	120.32	66.66
176	3	155.76	1.00	153.48	67.16
177	4	185.30	.68	124.21	67.48
178	0	0.00	0.00	-3.00	67.08
179	0	0.00	0.00	-3.00	66.69
180	5	108.78	.97	102.99	66.89
181	4	82.87	.78	61.96	66.87
182	2	68.33	.81	52.39	66.79
183	2	43.93	.99	40.75	66.65
184	2	92.28	.62	54.36	66.58
185	2	49.77	.72	32.94	66.40
186	1	10.66	.82	5.92	66.07
187	4	101.08	.80	78.87	66.14
188	3	84.32	1.00	81.58	66.22
189	3	116.27	.94	106.32	66.43
190	3	46.74	.77	33.63	66.26
191	4	63.46	.78	47.11	66.16
192	1	69.48	.75	49.50	66.07
193	2	33.12	.86	25.72	65.87
194	0	0.00	0.00	-3.00	65.51
195	3	127.19	.85	105.54	65.72
196	4	146.63	.94	135.76	66.07
197	2	48.18	.70	31.24	65.90
198	4	81.22	1.03	81.18	65.97
199	4	76.37	.78	57.05	65.93
200	2	46.29	.76	32.42	65.76
201	1	49.99	.54	24.21	65.55
202	2	56.42	.70	36.53	65.41
203	1	10.27	.81	5.52	65.12
204	2	33.89	.89	27.39	64.93
205	1	12.06	.78	6.59	64.65
206	3	67.32	.87	55.92	64.60
207	4	146.58	.90	128.85	64.91
208	3	78.10	.70	51.99	64.85
209	8	221.96	.71	155.14	65.28
210	3	32.79	.78	23.02	65.08
211	7	179.39	.62	109.95	65.30
212	4	102.93	.65	65.00	65.29
213	5	182.99	.59	106.45	65.49
214	2	23.19	.90	18.16	65.27
215	4	179.31	.85	149.58	65.66
216	3	40.05	.89	33.09	65.51
217	4	68.35	.79	51.88	65.44
218	4	94.78	.90	82.79	65.52
219	5	171.29	.97	164.52	65.98
220	0	0.00	0.00	-3.00	65.66
221	5	110.02	.92	98.94	65.81
222	2	42.10	.68	26.12	65.63
223	5	142.72	.95	132.74	65.94
224	5	132.25	.82	105.64	66.11
225	2	60.06	.70	39.54	65.99
226	2	16.24	.70	8.69	65.74
227	3	65.40	.94	58.74	65.71
228	3	72.30	1.04	72.30	65.74

229	4	72.06	.78	53.48	65.69
230	4	128.08	.89	111.51	65.89
231	3	45.47	.67	27.83	65.72
232	6	200.29	1.01	200.10	66.30
233	2	38.19	.75	25.98	66.13
234	2	67.98	.86	56.05	66.08
235	3	43.40	.76	30.58	65.93
236	4	121.19	.84	99.09	66.07
237	1	30.96	.73	19.73	65.88
238	4	70.22	.61	40.32	65.77
239	5	90.32	.66	57.49	65.74
240	0	0.00	0.00	-3.00	65.45
241	2	84.62	.92	75.16	65.49
242	2	63.96	.82	49.89	65.42
243	2	119.11	.63	72.76	65.45
244	3	75.18	.90	64.79	65.45
245	4	163.88	.85	136.17	65.74
246	2	62.32	.86	51.07	65.68
247	2	43.48	.80	32.27	65.55
248	4	82.25	.96	76.80	65.59
249	2	51.07	.74	35.23	65.47
250	2	35.34	.59	18.11	65.28
251	3	62.71	.98	59.10	65.26
252	1	15.34	.96	11.95	65.04
253	2	113.74	.80	88.76	65.14
254	2	62.76	.62	36.00	65.02
255	5	183.22	.76	137.64	65.31
256	4	51.97	1.08	53.87	65.26
257	2	32.48	.70	19.95	65.09
258	3	95.57	1.01	93.89	65.20
259	1	24.58	.86	18.21	65.02
260	4	100.22	.87	85.22	65.09
261	5	126.84	.96	119.65	65.30
262	1	21.42	.95	17.54	65.12
263	1	14.90	.85	9.85	64.91
264	5	81.39	.90	71.00	64.93
265	7	192.19	1.04	197.50	65.43
266	0	0.00	0.00	-3.00	65.18
267	3	166.32	.75	121.45	65.39
268	3	115.73	.76	85.21	65.46
269	3	237.11	.98	230.96	66.08
270	2	82.53	.93	73.67	66.11
271	2	27.87	.86	21.19	65.94
272	2	28.30	.77	19.11	65.77
273	1	10.30	.77	5.10	65.55
274	4	136.50	.84	111.76	65.71
275	4	97.33	.86	80.92	65.77
276	7	110.15	.77	83.08	65.83
277	3	117.57	.73	83.53	65.90
278	4	56.03	.70	36.79	65.79
279	2	66.87	.75	47.18	65.72
280	2	104.59	.65	65.00	65.72
281	5	81.22	.74	57.78	65.69
282	3	56.46	1.10	59.64	65.67
283	2	102.32	1.10	109.98	65.83
284	0	0.00	0.00	-3.00	65.59
285	2	135.46	.80	105.93	65.73
286	4	150.39	.57	84.00	65.79
287	1	20.62	.65	10.58	65.60
288	4	170.68	1.05	176.30	65.98
289	2	25.73	.69	15.04	65.81
290	1	39.28	1.05	38.54	65.71

291	3	111.95	.75	81.07	65.77
292	3	46.71	.90	39.59	65.68
293	2	148.66	.90	130.68	65.90
294	3	42.49	.54	20.28	65.74
295	4	72.96	.63	43.69	65.67
296	4	58.64	.75	41.72	65.59
297	1	65.29	.91	56.88	65.56
298	3	59.52	.95	54.12	65.52
299	2	41.19	.92	35.28	65.42
300	3	45.93	1.01	43.62	65.35
301	2	111.35	.64	68.44	65.36
302	3	81.95	.82	64.63	65.35
303	2	9.02	.91	5.48	65.16
304	2	25.66	.93	21.17	65.01
305	5	208.44	.59	121.06	65.20
306	6	178.62	.85	149.63	65.47
307	4	58.22	.89	49.17	65.42
308	6	305.88	.84	254.27	66.03
309	5	318.55	.88	279.47	66.72
310	3	58.94	1.01	56.88	66.69
311	2	9.99	.87	6.03	66.50
312	4	144.74	.81	114.40	66.65
313	7	290.35	.88	252.48	67.24
314	5	90.82	.89	78.80	67.28
315	2	90.02	.95	82.82	67.33
316	3	46.78	.59	25.14	67.20
317	3	66.56	.77	48.68	67.14
318	5	71.21	.93	64.33	67.13
319	1	35.39	.75	23.61	66.99
320	4	56.28	.89	47.70	66.93
321	1	21.59	.94	17.49	66.78
322	4	77.44	.93	69.72	66.79
323	2	12.08	.77	6.66	66.60
324	2	111.91	.82	88.91	66.67
325	2	33.23	.88	26.46	66.55
326	3	130.08	.93	117.92	66.70
327	0	0.00	0.00	-3.00	66.49
328	2	36.83	.78	26.00	66.37
329	4	198.73	.92	179.64	66.71
330	6	127.67	.89	112.09	66.85
331	4	81.49	.95	75.09	66.87
332	1	10.50	.98	7.39	66.69
333	4	135.69	.81	107.22	66.82
334	5	110.81	1.01	109.48	66.94
335	3	50.27	.80	37.68	66.86
336	2	150.83	.88	129.77	67.04
337	5	109.13	.88	93.81	67.12
338	5	107.49	.82	86.15	67.18
339	5	52.73	.62	30.55	67.07
340	6	116.25	1.12	128.41	67.25
341	1	24.41	.81	16.91	67.10
342	3	47.24	.80	35.45	67.01
343	4	164.77	.96	156.25	67.27
344	4	165.69	.84	137.06	67.47
345	5	210.82	.80	166.73	67.76
346	4	140.86	.66	90.53	67.83
347	5	118.45	.57	65.08	67.82
348	3	39.52	1.05	38.94	67.74
349	2	135.67	.89	117.93	67.88
350	3	45.99	.85	36.44	67.79
351	4	56.88	.73	39.31	67.71
352	3	53.83	.79	39.72	67.63

353	2	36.15	.97	32.44	67.53
354	3	85.85	.73	60.06	67.51
355	2	63.40	.81	48.92	67.46
356	1	23.07	.76	14.67	67.31
357	2	24.40	1.04	22.73	67.18
358	4	233.50	1.02	235.80	67.65
359	2	40.20	.86	31.75	67.55
360	2	99.04	.79	75.22	67.58
361	1	10.86	.72	4.93	67.40
362	3	42.47	.91	35.93	67.32
363	1	140.00	.82	111.65	67.44
364	3	31.09	.89	25.06	67.32
365	5	160.49	.85	133.43	67.50
366	1	11.79	.99	8.79	67.34
367	3	116.60	.85	96.71	67.42
368	4	114.60	.84	93.43	67.49
369	2	19.36	.82	13.08	67.35
370	5	148.53	1.08	158.49	67.59
371	5	118.97	1.00	116.98	67.72
372	5	75.39	1.00	72.85	67.74
373	3	123.28	.92	111.40	67.86
374	5	229.61	1.05	239.13	68.31
375	5	89.33	.85	73.77	68.33
376	3	66.18	.78	48.92	68.28
377	2	19.08	.84	13.36	68.13
378	2	40.68	.82	30.75	68.03
379	1	5.27	.82	1.45	67.86
380	4	72.81	.77	53.90	67.82
381	2	50.20	.66	30.58	67.72
382	1	30.01	1.04	28.24	67.62
383	1	6.09	1.03	3.40	67.45
384	4	52.00	.99	48.82	67.40
385	2	67.27	.84	54.11	67.37
386	1	10.25	.95	6.87	67.21
387	2	97.62	.80	74.98	67.23
388	6	110.19	.84	90.93	67.29
389	6	96.36	.87	81.36	67.33
390	2	70.42	.64	42.34	67.26
391	3	125.93	.73	89.71	67.32
392	4	60.80	.80	46.07	67.27
393	1	26.17	.72	15.86	67.14
394	4	71.05	.81	55.27	67.11
395	4	139.19	.89	121.76	67.24
396	1	10.88	.51	2.70	67.08
397	4	157.85	.92	142.81	67.27
398	1	7.56	.80	3.23	67.11
399	3	56.47	.87	46.54	67.06
400	7	187.61	.74	136.74	67.23
401	3	93.10	.88	79.02	67.26
402	1	7.09	.90	3.54	67.11
403	1	21.43	.68	11.81	66.97
404	4	106.34	1.01	104.92	67.06
405	3	67.93	1.03	67.44	67.06
406	2	61.13	.67	38.03	66.99
407	5	106.43	.88	91.25	67.05
408	1	118.99	.85	98.09	67.13
409	1	14.24	1.10	12.89	66.99
410	5	267.10	.53	138.32	67.17
411	4	68.07	.89	58.28	67.15
412	1	34.85	.90	28.44	67.05
413	0	0.00	0.00	-3.00	66.88
414	1	7.86	.82	3.58	66.73

415	2	22.39	1.03	20.25	66.62
416	2	103.76	.86	86.78	66.67
417	3	69.77	.61	39.79	66.60
418	3	132.76	.57	72.91	66.62
419	1	31.52	.97	27.65	66.52
420	3	152.71	.90	134.40	66.69
421	1	44.28	.94	38.83	66.62
422	2	54.03	.90	46.04	66.57
423	2	74.81	.95	68.51	66.58
424	1	11.22	1.02	8.65	66.44
425	3	73.13	.92	64.67	66.43
426	3	50.83	.92	44.42	66.38
427	2	22.22	.95	18.42	66.27
428	4	79.78	.80	61.41	66.26
429	4	42.77	.89	35.64	66.19
430	4	189.88	.90	169.43	66.43
431	4	98.89	.58	55.18	66.40
432	4	199.46	.85	166.82	66.63
433	5	187.53	.84	154.50	66.84
434	0	0.00	0.00	-3.00	66.68
435	1	6.00	.66	1.09	66.53
436	4	62.52	.92	55.09	66.50
437	3	89.47	.63	53.66	66.47
438	1	9.12	.95	5.78	66.33
439	2	120.28	.85	99.50	66.41
440	4	73.50	.95	67.59	66.41
441	4	115.15	.74	83.18	66.45
442	2	35.23	.95	30.77	66.37
443	1	34.12	.66	19.58	66.26
444	3	62.27	.62	35.85	66.19
445	2	63.79	.77	46.49	66.15
446	4	95.88	.86	79.71	66.18
447	3	77.92	.89	66.96	66.18
448	6	83.44	.92	74.78	66.20
449	4	178.90	1.08	190.81	66.48
450	4	37.46	.91	31.83	66.40
451	4	113.76	.67	73.42	66.42
452	2	35.21	.92	29.71	66.34
453	3	34.86	.76	23.84	66.24
454	0	0.00	0.00	-3.00	66.09
455	3	36.49	.79	26.43	66.00
456	2	29.00	1.00	26.39	65.91
457	2	42.32	.87	34.29	65.85
458	1	45.82	.61	25.04	65.76
459	2	33.20	.73	21.37	65.66
460	1	20.75	.75	12.61	65.54
461	3	73.17	.93	65.66	65.54
462	2	94.48	.82	74.67	65.56
463	3	19.87	.70	11.41	65.45
464	3	44.26	.80	32.87	65.38
465	3	52.04	.58	27.67	65.30
466	4	66.38	.98	62.90	65.29
467	2	110.36	.84	89.58	65.34
468	2	55.83	.95	50.31	65.31
469	3	28.97	.87	22.62	65.22
470	2	53.71	.57	27.86	65.14
471	3	62.70	1.00	60.08	65.13
472	3	89.33	.86	74.57	65.15
473	4	109.69	1.14	122.40	65.27
474	1	45.57	.82	34.37	65.21
475	6	184.99	1.05	192.34	65.47
476	6	173.40	.58	98.35	65.54

477	2	24.17	.98	20.87	65.45
478	4	148.12	.90	130.87	65.59
479	1	103.47	.81	81.26	65.62
480	4	108.22	.80	84.61	65.66
481	3	106.87	.73	75.05	65.68
482	3	112.58	.71	77.64	65.70
483	8	288.05	.92	262.33	66.11
484	5	109.23	.97	103.84	66.19
485	2	59.56	.78	44.00	66.14
486	4	137.82	.79	106.57	66.22
487	1	27.97	.78	18.97	66.13
488	2	33.00	.71	20.69	66.03
489	5	114.15	.82	91.45	66.09
490	2	56.81	.90	48.46	66.05
491	1	62.34	1.01	60.13	66.04
492	3	111.68	.62	66.63	66.04
493	2	50.07	.99	46.68	66.00
494	0	0.00	0.00	-3.00	65.86
495	2	40.28	.85	31.49	65.79
496	4	105.05	.81	82.78	65.83
497	2	28.71	.83	21.11	65.74
498	1	29.00	.75	19.01	65.64
499	6	146.57	.87	124.70	65.76
500	4	37.61	.96	33.63	65.70
501	1	16.72	.71	9.02	65.58
502	4	102.21	1.01	101.24	65.65
503	3	138.94	.90	121.90	65.77
504	2	32.87	.88	26.18	65.69
505	0	0.00	0.00	-3.00	65.55
506	1	32.68	.63	17.58	65.46
507	3	93.82	.95	86.49	65.50
508	5	113.91	.87	97.26	65.56
509	5	128.46	1.05	132.44	65.69
510	3	39.01	.89	32.13	65.63
511	4	57.16	.62	33.21	65.56
512	4	87.05	.82	69.39	65.57
513	4	115.79	.95	107.38	65.65
514	3	208.52	.62	126.81	65.77
515	0	0.00	0.00	-3.00	65.64
516	1	7.95	.88	4.19	65.52
517	4	85.46	.93	77.36	65.54
518	3	38.04	.85	29.94	65.47
519	4	158.75	.72	111.72	65.56
520	3	78.77	.74	56.01	65.54
521	3	72.31	.67	45.95	65.50
522	3	68.79	.87	57.08	65.49
523	3	37.86	.88	30.73	65.42
524	6	313.07	.81	252.53	65.78
525	5	98.16	.79	75.45	65.80
526	3	77.27	.80	59.57	65.79
527	4	92.44	.72	64.36	65.78
528	1	27.76	1.01	25.11	65.71
529	3	67.43	.80	51.34	65.68
530	4	83.24	.89	71.36	65.69
531	2	26.65	.81	18.77	65.60
532	2	40.25	.87	32.26	65.54
533	3	121.70	1.08	128.81	65.66
534	2	20.41	.79	13.37	65.56
535	3	47.32	.85	37.79	65.51
536	2	12.13	.56	4.14	65.39
537	4	49.13	.92	42.68	65.35
538	5	107.41	.81	85.14	65.39

539	5	178.37	.84	148.14	65.54
540	1	37.40	1.01	34.84	65.48
541	4	106.78	1.00	104.59	65.56
542	0	0.00	0.00	-3.00	65.43
543	5	278.58	.95	262.42	65.79
544	2	47.47	.68	29.36	65.73
545	1	9.88	.90	6.08	65.62
546	3	72.37	.88	60.87	65.61
547	2	80.53	.79	60.89	65.60
548	4	76.68	.76	55.59	65.58
549	1	67.69	.80	51.36	65.55
550	4	66.83	.83	53.22	65.53
551	1	106.73	.71	72.52	65.55
552	8	210.09	.97	202.99	65.79
553	4	166.72	.54	87.20	65.83
554	4	26.69	.71	16.55	65.74
555	6	109.19	.81	86.74	65.78
556	3	248.15	.84	205.05	66.03
557	6	134.80	.78	103.28	66.10
558	4	137.87	.79	106.29	66.17
559	0	0.00	0.00	-3.00	66.05
560	4	106.31	.63	65.09	66.05
561	6	196.43	1.02	197.43	66.28
562	4	54.98	.87	45.43	66.24
563	5	153.38	.70	105.25	66.31
564	3	117.47	.93	106.93	66.38
565	4	127.93	.64	79.72	66.41
566	5	120.56	.95	112.29	66.49
567	7	177.64	.91	159.15	66.65
568	3	75.03	.75	54.00	66.63
569	4	100.80	.82	80.73	66.65
570	1	20.18	.91	15.53	66.57
571	5	102.92	.66	65.89	66.56
572	1	13.92	.76	7.69	66.46
573	4	97.99	.98	93.16	66.51
574	2	15.97	.86	10.97	66.41
575	3	124.96	.78	94.43	66.46
576	2	22.45	.74	13.92	66.37
577	4	64.07	.95	58.45	66.35
578	6	166.34	.92	150.32	66.50
579	2	17.73	.95	14.22	66.41
580	2	89.84	.78	67.21	66.41
581	4	51.43	.89	43.31	66.37
582	3	51.92	.95	46.90	66.34
583	3	63.49	.59	35.18	66.28
584	1	7.46	.97	4.38	66.18
585	4	171.19	.66	109.94	66.25
586	3	77.64	.89	66.39	66.25
587	2	43.32	.73	29.14	66.19
588	2	17.49	1.00	14.83	66.10
589	2	12.60	.90	8.69	66.01
590	4	103.57	.87	87.85	66.04
591	4	102.03	.91	90.90	66.08
592	1	23.09	.76	14.59	66.00
593	2	49.22	.99	45.81	65.96
594	1	2.50	.95	-.47	65.85
595	2	31.00	.84	23.25	65.78
596	4	220.62	1.01	219.36	66.04
597	5	262.40	1.06	275.06	66.39
598	2	173.87	.88	149.49	66.53
599	2	32.36	1.14	34.18	66.47
600	4	58.80	.75	41.80	66.43

601	4	134.82	.99	131.67	66.54
602	3	77.80	.74	54.89	66.52
603	5	368.84	.98	358.84	67.01
604	3	71.46	.76	51.71	66.98
605	4	151.47	.78	116.31	67.06
606	3	49.01	.89	40.90	67.02
607	1	17.00	.68	8.72	66.92
608	2	48.04	.99	44.63	66.89
609	3	57.72	.90	49.27	66.86
610	2	40.85	.95	36.05	66.81
611	4	115.94	.80	90.61	66.85
612	2	25.32	1.05	23.90	66.78
613	3	55.54	.55	27.84	66.71
614	4	129.16	.99	125.02	66.81
615	2	203.53	.95	190.37	67.01
616	2	35.51	.98	32.25	66.95
617	3	77.09	.75	55.62	66.93
618	3	100.85	.97	95.55	66.98
619	7	121.11	.84	99.86	67.03
620	4	81.24	.82	63.85	67.03
621	0	0.00	0.00	-3.00	66.91
622	3	46.15	.57	23.86	66.84
623	6	221.94	.65	142.19	66.97
624	6	204.77	.80	161.33	67.12
625	3	71.42	.99	68.41	67.12
626	4	255.34	.70	175.40	67.29
627	3	129.58	1.04	132.65	67.40
628	1	62.61	.77	45.06	67.36
629	3	66.29	.57	35.45	67.31
630	4	83.38	1.00	81.11	67.33
631	3	99.04	.87	84.00	67.36
632	2	30.73	.84	23.13	67.29
633	2	112.42	.84	91.97	67.33
634	5	125.52	.65	79.80	67.35
635	8	218.03	.84	180.79	67.53
636	2	18.54	1.01	15.98	67.44
637	2	62.81	.71	42.18	67.41
638	1	14.49	.76	8.16	67.31
639	3	176.52	1.03	180.00	67.49
640	3	105.78	.76	77.70	67.50
641	1	109.74	.93	98.71	67.55
642	3	59.08	.69	38.15	67.51
643	2	36.05	.91	30.11	67.45
644	5	123.03	.74	88.91	67.48
645	0	0.00	0.00	-3.00	67.37
646	3	51.05	.79	37.81	67.33
647	4	90.91	1.03	91.12	67.36
648	2	26.46	.94	22.21	67.29
649	2	39.16	.97	35.24	67.25
650	3	46.08	.76	32.60	67.19
651	3	84.28	.94	77.04	67.21
652	2	24.75	.89	19.27	67.13
653	6	84.86	.66	54.14	67.11
654	4	105.29	.76	77.13	67.13
655	1	29.82	.80	21.06	67.06
656	3	40.81	.95	36.16	67.01
657	4	255.32	.88	221.15	67.25
658	2	65.67	.86	53.67	67.23
659	2	33.75	.61	17.90	67.15
660	1	35.03	1.01	32.47	67.10
661	6	216.11	.64	135.94	67.20
662	4	129.31	.69	86.98	67.23

663	4	106.15	1.03	106.92	67.29
664	2	32.78	.84	24.79	67.23
665	7	152.33	1.03	154.63	67.36
666	4	83.78	.92	74.91	67.37
667	3	134.85	.97	128.37	67.46
668	2	53.67	.80	40.09	67.42
669	2	78.43	.86	64.49	67.42
670	4	55.03	1.00	52.62	67.39
671	3	114.17	.80	88.82	67.43
672	2	40.13	.53	18.45	67.35
673	4	69.77	.89	59.79	67.34
674	1	27.81	.82	20.07	67.27
675	2	19.20	.88	14.20	67.19
676	3	56.07	.97	51.60	67.17
677	4	135.30	.82	108.31	67.23
678	3	66.58	.68	42.48	67.19
679	4	50.25	.85	40.38	67.16
680	5	145.83	.84	120.27	67.23
681	1	56.21	.83	43.59	67.20
682	3	131.39	.87	111.34	67.26
683	0	0.00	0.00	-3.00	67.16
684	5	152.17	.73	108.42	67.22
685	4	181.01	.82	145.26	67.33
686	4	96.48	1.00	94.43	67.37
687	2	56.78	.88	47.34	67.35
688	6	145.81	.72	102.90	67.40
689	3	110.81	.72	77.17	67.41
690	2	36.83	1.00	34.23	67.36
691	6	180.77	.88	157.78	67.49
692	1	28.27	.88	22.08	67.43
693	4	88.48	.92	78.93	67.44
694	3	97.26	.72	67.83	67.45
695	2	32.64	.84	24.56	67.38
696	4	146.85	.71	101.38	67.43
697	2	117.85	.90	103.15	67.48
698	4	93.86	.76	68.56	67.49
699	0	0.00	0.00	-3.00	67.38
700	2	51.79	.75	36.26	67.34
701	5	78.39	.64	47.63	67.31
702	3	84.02	.77	61.86	67.30
703	4	87.68	.95	80.55	67.32
704	3	97.17	.55	50.91	67.30
705	2	26.85	.78	18.35	67.23
706	2	42.73	.87	34.54	67.18
707	2	69.03	.81	53.51	67.16
708	1	30.12	.89	24.04	67.10
709	3	37.92	.76	26.40	67.05
710	3	75.44	1.01	73.72	67.06
711	4	79.03	1.01	77.34	67.07
712	4	92.93	.86	77.93	67.09
713	3	114.75	1.11	124.45	67.17
714	2	34.27	.84	26.01	67.11
715	5	71.37	.77	52.99	67.09
716	4	160.63	.69	108.13	67.15
717	7	156.55	.59	91.02	67.18
718	1	42.96	.97	38.62	67.14
719	7	186.02	.84	154.07	67.26
720	4	71.17	.54	35.74	67.22
721	4	98.80	1.05	100.89	67.26
722	2	16.43	.73	9.30	67.18
723	4	144.47	1.11	157.99	67.31
724	1	73.36	.61	41.82	67.27

725	3	65.96	.88	55.81	67.26
726	0	0.00	0.00	-3.00	67.16
727	2	76.36	.66	47.83	67.13
728	1	8.99	.74	3.81	67.05
729	4	106.85	.95	99.31	67.09
730	2	93.35	1.12	102.01	67.14
731	3	106.67	.94	98.06	67.18
732	3	49.35	.89	41.37	67.15
733	2	73.45	.83	58.36	67.13
734	2	58.83	.95	53.34	67.12
735	5	160.54	.71	111.90	67.18
736	3	70.26	.62	41.19	67.14
737	2	145.34	.98	140.01	67.24
738	2	90.10	.97	85.12	67.26
739	3	175.07	.75	128.82	67.35
740	3	48.96	.75	33.94	67.30
741	2	39.34	1.02	37.36	67.26
742	4	82.56	1.02	81.44	67.28
743	5	147.74	.95	137.67	67.38
744	3	88.55	.67	57.21	67.36
745	2	63.70	.77	46.53	67.33
746	2	17.27	.90	12.78	67.26
747	3	49.12	.93	43.22	67.23
748	0	0.00	0.00	-3.00	67.13
749	4	108.80	.72	75.85	67.15
750	6	107.57	.67	70.47	67.15
751	3	35.90	.73	23.64	67.09
752	2	20.87	.63	10.44	67.02
753	5	133.84	1.08	141.96	67.12
754	5	186.90	.86	158.81	67.24
755	3	64.72	.67	40.67	67.20
756	5	99.22	1.12	108.39	67.26
757	4	127.06	1.06	132.54	67.34
758	5	213.03	.93	195.71	67.51
759	7	224.39	.83	184.26	67.67
760	2	100.55	.89	87.13	67.69
761	5	150.55	1.05	156.19	67.81
762	1	55.21	.68	34.80	67.77
763	2	161.29	.85	134.61	67.85
764	4	165.35	.75	121.49	67.92
765	3	66.71	.78	49.30	67.90
766	1	11.03	.95	7.65	67.82
767	3	29.95	.83	22.37	67.76
768	3	50.62	.99	47.72	67.74
769	1	14.84	1.05	12.71	67.66
770	2	98.57	.95	91.34	67.69
771	6	241.27	.70	166.65	67.82
772	3	102.70	1.06	105.95	67.87
773	5	179.70	.66	116.54	67.94
774	2	25.59	.98	22.49	67.88
775	5	139.35	.77	105.24	67.92
776	4	51.32	1.05	51.30	67.90
777	2	126.96	.93	115.05	67.96
778	3	98.31	1.08	103.50	68.01
779	1	5.87	.97	2.84	67.93
780	4	49.56	.74	34.43	67.88
781	3	125.36	.82	99.70	67.92
782	4	97.79	.74	70.35	67.93
783	1	26.65	.77	17.77	67.86
784	5	53.65	.76	38.26	67.83
785	3	48.13	.72	32.15	67.78
786	4	50.09	.92	43.57	67.75

787	5	268.35	.69	183.17	67.90
788	5	52.16	.86	42.73	67.86
789	7	227.35	.58	129.15	67.94
790	3	123.37	.70	83.23	67.96
791	6	220.95	.99	216.57	68.15
792	4	77.59	.88	65.76	68.15
793	0	0.00	0.00	-3.00	68.06
794	3	62.05	.85	49.93	68.03
795	3	149.27	.67	97.15	68.07
796	3	64.55	.72	44.05	68.04
797	7	196.83	.79	154.07	68.15
798	3	39.87	1.02	38.02	68.11
799	2	121.40	.65	76.52	68.12
800	3	81.34	.91	71.68	68.12
801	0	0.00	0.00	-3.00	68.04
802	4	64.46	.78	48.03	68.01
803	3	35.98	1.05	35.22	67.97
804	2	32.20	.72	20.47	67.91
805	3	59.10	.78	43.69	67.88
806	3	107.40	.58	59.73	67.87
807	4	100.10	.83	80.88	67.89
808	2	42.48	.74	28.83	67.84
809	2	22.94	.72	13.76	67.77
810	4	143.80	.98	139.14	67.86
811	1	39.39	.78	28.00	67.81
812	4	121.03	.84	99.36	67.85
813	1	7.69	.89	4.03	67.77
814	4	102.77	.86	86.06	67.79
815	3	150.14	.89	130.44	67.87
816	2	53.61	.53	25.83	67.82
817	2	101.02	.91	89.00	67.84
818	4	129.17	1.04	131.75	67.92
819	2	41.61	.84	32.13	67.88
820	3	103.07	.84	84.30	67.90
821	6	112.40	.67	73.00	67.91
822	4	135.28	.86	114.57	67.96
823	5	148.40	.84	123.12	68.03
824	1	45.92	1.01	43.66	68.00
825	3	147.20	.84	121.28	68.06
826	7	160.11	.96	152.26	68.17
827	7	310.18	.93	286.23	68.43
828	4	82.17	.98	77.82	68.44
829	4	115.43	.79	89.25	68.47
830	1	10.69	.95	7.29	68.39
831	2	143.61	1.09	154.52	68.50
832	2	121.62	.74	87.14	68.52
833	5	141.98	.82	113.49	68.57
834	4	153.20	.76	113.75	68.63
835	0	0.00	0.00	-3.00	68.54
836	5	214.81	.91	193.06	68.69
837	1	16.05	.86	11.00	68.62
838	1	80.43	.79	60.80	68.61
839	1	26.99	.59	12.99	68.55
840	3	35.92	.81	26.60	68.50
841	6	119.20	.90	105.59	68.54
842	2	17.49	.56	7.15	68.47
843	3	99.31	1.07	103.34	68.51
844	1	15.99	.97	12.70	68.44
845	4	133.08	.85	111.29	68.49
846	3	80.97	.87	68.18	68.49
847	2	16.01	.99	13.22	68.43
848	4	75.06	.65	46.46	68.40

849	4	76.49	.88	65.20	68.40
850	3	131.13	1.08	139.69	68.48
851	5	55.97	1.02	54.67	68.47
852	3	175.05	.85	145.97	68.56
853	5	148.64	.64	93.43	68.59
854	4	155.51	.75	114.18	68.64
855	4	99.10	1.02	98.70	68.67
856	2	61.57	.68	39.06	68.64
857	1	45.98	.79	33.65	68.60
858	2	42.86	.77	30.35	68.55
859	4	57.38	.73	39.48	68.52
860	5	278.61	.80	219.98	68.70
861	1	7.82	.71	2.71	68.62
862	2	21.67	.72	12.95	68.55
863	5	89.49	.97	84.41	68.57
864	3	132.26	.90	115.94	68.63
865	1	16.65	.88	11.73	68.56
866	1	75.61	.65	46.41	68.54
867	2	30.31	.63	16.38	68.48
868	1	59.10	.90	50.14	68.46
869	3	51.81	.80	38.95	68.42
870	2	78.80	.82	62.07	68.41
871	3	35.14	.85	27.39	68.37
872	4	42.67	.89	35.67	68.33
873	4	92.72	.69	61.14	68.32
874	1	25.99	1.09	25.58	68.27
875	1	7.83	.55	1.49	68.20
876	3	98.31	.79	74.79	68.20
877	1	60.85	.72	41.18	68.17
878	2	82.45	.91	72.05	68.18
879	4	46.22	.64	27.37	68.13
880	7	105.05	1.04	107.74	68.18
881	4	72.59	.61	41.54	68.15
882	2	131.70	.98	126.04	68.21
883	3	42.01	1.03	40.67	68.18
884	2	55.18	.95	49.75	68.16
885	2	49.31	.62	27.65	68.11
886	2	33.63	1.07	33.22	68.07
887	4	123.45	.84	101.33	68.11
888	2	42.31	1.12	44.73	68.09
889	7	133.15	.91	119.60	68.14
890	3	118.14	.90	104.31	68.18
891	3	117.93	.99	113.85	68.23
892	5	175.85	.88	153.03	68.33
893	4	89.84	.78	67.39	68.33
894	1	14.98	1.04	12.70	68.27
895	3	60.13	.79	45.10	68.24
896	2	76.65	.60	43.22	68.21
897	2	36.35	.92	30.89	68.17
898	1	18.17	1.11	17.27	68.11
899	3	60.94	.94	54.87	68.10
900	2	69.61	.96	63.87	68.10
901	4	165.45	.61	98.87	68.13
902	2	44.09	.79	32.31	68.09
903	4	121.40	.81	95.86	68.12
904	2	32.84	.99	29.82	68.08
905	2	88.36	.91	77.91	68.09
906	1	6.56	.65	1.42	68.02
907	6	299.06	.92	274.07	68.24
908	2	67.80	.90	58.48	68.23
909	4	98.84	.71	67.79	68.23
910	3	64.58	.96	59.49	68.22

911	2	64.88	1.01	62.52	68.22
912	3	75.28	1.11	81.36	68.23
913	4	120.56	.95	112.32	68.28
914	4	103.66	.79	79.48	68.29
915	2	59.12	.90	50.52	68.27
916	2	106.89	.95	98.41	68.30
917	2	117.69	.98	112.95	68.35
918	5	91.22	.96	84.89	68.37
919	2	120.53	.68	78.87	68.38
920	4	71.18	.69	46.95	68.36
921	2	63.55	.97	58.82	68.35
922	4	41.41	.79	30.41	68.31
923	3	61.95	.74	43.02	68.28
924	1	17.27	.64	8.23	68.21
925	2	28.94	1.04	27.33	68.17
926	3	57.08	1.03	56.27	68.16
927	4	160.68	.93	146.55	68.24
928	2	44.95	.86	35.79	68.21
929	4	57.39	.67	36.20	68.17
930	6	285.64	.76	213.59	68.33
931	2	18.85	.76	11.59	68.27
932	5	131.95	1.01	131.12	68.34
933	1	24.78	1.05	23.11	68.29
934	4	149.44	.82	119.74	68.34
935	2	25.00	.83	18.00	68.29
936	2	108.64	.81	85.45	68.31
937	6	127.18	.60	73.65	68.31
938	1	34.01	1.01	31.54	68.27
939	3	68.04	.76	49.10	68.25
940	4	114.78	.83	92.66	68.28
941	3	93.84	.74	66.81	68.28
942	1	12.16	.90	8.12	68.21
943	6	235.57	.91	212.77	68.37
944	5	141.75	.90	126.02	68.43
945	4	86.68	.91	76.62	68.44
946	1	95.48	.83	76.82	68.45
947	1	12.50	1.00	9.69	68.38
948	3	94.18	.82	74.29	68.39
949	3	131.53	.81	104.49	68.43
950	2	46.95	.94	41.47	68.40
951	0	0.00	0.00	-3.00	68.32
952	2	41.66	1.00	39.17	68.29
953	5	107.98	.69	72.56	68.30
954	3	112.16	.81	88.83	68.32
955	2	58.17	1.00	55.31	68.31
956	1	18.07	.73	10.31	68.25
957	4	226.90	.62	139.19	68.32
958	3	71.78	.93	64.04	68.31
959	2	18.57	.81	12.40	68.26
960	2	120.67	1.09	128.42	68.32
961	2	74.21	.80	56.32	68.31
962	2	9.77	.94	6.46	68.24
963	3	102.85	.81	80.43	68.26
964	2	48.64	.67	29.81	68.22
965	4	134.03	.88	115.00	68.26
966	4	141.04	1.07	148.36	68.35
967	2	58.36	.94	52.05	68.33
968	3	43.71	.90	36.96	68.30
969	4	245.90	.76	184.27	68.42
970	4	118.89	.92	106.62	68.46
971	3	81.39	.95	74.41	68.46
972	3	99.31	.67	63.99	68.46

973	3	75.64	1.01	74.07	68.46
974	5	68.62	.97	64.64	68.46
975	2	49.27	.83	37.95	68.43
976	4	120.25	.97	114.25	68.48
977	4	113.96	.71	78.62	68.49
978	1	9.93	.97	6.76	68.42
979	1	23.98	.83	17.12	68.37
980	0	0.00	0.00	-3.00	68.30
981	3	71.60	.80	54.54	68.28
982	3	36.94	.91	30.93	68.25
983	2	48.01	.67	29.35	68.21
984	2	59.59	.74	41.24	68.18
985	2	91.15	.59	50.75	68.16
986	1	7.20	1.10	5.07	68.10
987	3	89.59	.80	69.07	68.10
988	1	33.57	.63	18.46	68.05
989	2	50.60	1.07	51.47	68.03
990	2	96.38	.74	68.50	68.03
991	4	47.68	.82	36.88	68.00
992	3	91.39	1.01	89.55	68.02
993	3	30.18	.97	26.77	67.98
994	7	170.25	.82	138.48	68.05
995	3	54.44	.77	39.36	68.02
996	8	237.43	.73	172.56	68.13
997	1	13.36	.99	10.38	68.07
998	3	61.66	.58	33.07	68.03
999	3	112.05	.90	98.59	68.06
1000	3	43.11	.63	24.54	68.02

THE BEST ESTIMATE OF PROFIT BY THE MONTE CARLO SIMULATION METHOD
AT 1000 ITERATIONS IS 68.02 MILLION DOLLARS

THE VARIANCE IS 2879.367
THE STANDARD DEVIATION IS 53.660
THE COEFFICIENT OF VARIATION IS .7889

PROFIT DISTRIBUTION OF AN EXPLORATORY DRILLING PROGRAM

NO. OF WELLS DRILLED	PROB OF SUCCESS	DRY HOLE COST	QMEDIAN	Q84	QMAX	QMIN
20	.150	.150	20.00	46.00	140.00	1.00

THE TRIANGULAR DISTRIBUTION FOR DETERMINATION OF THE PROFIT PER BARREL HAS THE FOLLOWING EXTREMES IN CENTS -
A = 50.00 B = 88.00 C = 115.00

UPPER LIMIT OF PROFIT INTERVAL	PROBABILITY OF INTERVAL	CUMMULATIVE PROBABILITY
-10	*0000	*0000
0	.0420	.0420
10	.0530	.0950
20	.0740	.1690
30	.0910	.2600
40	.1060	.3660
50	.0870	.4530
60	.0780	.5310
70	.0630	.5940
80	.0630	.6570
90	.0650	.7220
100	.0450	.7670
110	.0490	.8160
120	.0340	.8500
130	.0280	.8780
140	.0280	.9060
150	.0130	.9190
160	.0220	.9410
170	.0090	.9500
180	.0050	.9550
190	.0110	.9660
200	.0070	.9730
210	.0050	.9780
220	.0050	.9830
230	.0030	.9860
240	.0030	.9890
250	*0000	.9890
260	.0040	.9930
270	.0020	.9950
280	.0030	.9980
290	.0010	.9990
300	*0000	.9990
350	*0000	.9990
400	.0010	1.0000
450	*0000	1.0000
500	*0000	1.0000
550	*0000	1.0000

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VITA

Lawrence William Vogel was born in Brooklyn, New York on January 5, 1938, the son of Charles and Lillian Vogel. He attended the Brooklyn Technical High School and the City College of the University of the City of New York where he was awarded a Bachelor of Civil Engineering degree in June 1959.

After graduation and until entering OCS in November 1959, he was employed as a Civil Engineer with the M.W. Kellogg Company, New York. He was commissioned Ensign, in April 1960 and currently holds the rank of Lieutenant Commander, Civil Engineer Corps, U.S. Navy.

Subsequent to his commissioning, LCDR Vogel was assigned as Project Management Officer and then as Engineering Officer at the Public Works Department, Yokosuka, Japan, with additional duty in Contract Administration. Following this three year assignment, LCDR Vogel was ordered to the Public Works Center, Newport, Rhode Island, where he served in a facilities management position. For his next assignment, he served as Assistant Resident Officer in Chagre of Construction, Newport. Following this tour of duty he served as Staff Civil Engineer, U.S. Naval Air Station Norfolk, Virginia and in June 1967 was ordered to the Construction Directorate, U.S. Military Assistance Command, Vietnam. While serving with MACV, LCDR Vogel was assigned to the Base Development Division where he was responsible for construction planning, determining construction requirements and for monitoring all construction in

the I Corps Tactical Zone for all Free World Military Forces. Following his tour in Vietnam, he was assigned to the Civil Engineer Corps Officers School, Port Hueneme, California, first as Director, Contract Administration Division, and then in April 1970, as Executive Officer of the School.

In July 1970, he entered the Graduate School at The University of Texas at Austin to begin his work toward the degree of Master of Science in Petroleum Engineering.

LCDR Vogel is a registered Professional Engineer in the State of New York and is a member of the National Society of Professional Engineers, Society of American Military Engineers, Society of Petroleum Engineers (AIME) and Pi Epislon Tau honorary fraternity.

He was married in 1959 to the former Linda Lois Altshuler, of Brooklyn, New York. They have three children: Michael, age 10; Sharon, age 9; and Robin, age 7.

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This thesis was typed by Linda L. Vogel

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